Abstract—Thermoelectric generators (TEGs) are resistive dc sources that supply the most power when their resistance drops half the dc voltage. Not all this power reaches the load, however. The energy-harvesting circuit that draws and transfers this power loses some of it in the form of heat. These ohmic losses scale linearly and quadratically with current. From the perspective of the load, linear losses reduce the dc voltage of the source and quadratic losses raise the series resistance. This not only decreases the power delivered to the load but also shifts the maximum power point of the system. Neglecting this shift in maximum power point sacrifices 2.5% to 90% of the maximum achievable power when the added resistance is 10% to 50% of the thermoelectric resistance and the voltage loss is 5% to 23% of the thermoelectric voltage. This paper derives, models, and shows how the maximum power point of the thermoelectric harvester differs from that of the TEG and when that difference is significant.

Index Terms—TE, MPP, PIN, PO, PLOSS, Energy Harvester, Switched Inductor, DC voltage source, Source resistance, CMOS, MPPLOSS, MPP model, DC-DC power supply, Boost, ηC.

I. THERMOELECTRIC-POWERED SYSTEMS

Internet of Things (IoT) devices offer numerous benefits and functionalities. For instance, wireless sensors can monitor the production line by sensing chemicals [1]-[2]. However, these devices need to operate for months, and the use of large batteries can compromise their convenience. Recharging is also not a feasible option since most IoT devices are in hard-to-reach positions. Fortunately, thermoelectric generators (TEGs) provide solutions by generating voltage from thermal energy.

TEGs are low-power devices that can be modeled as a voltage source with a source resistance, as shown in Fig. 1. The voltage source generates $v_T$ in the range of 40 mV to 400 mV, with $\Delta T$ ranging from 1-5 K. The source resistance $R_T$ ranges from 1 $\Omega$ to 1.3 $\Omega$ [3]. Due to the low $v_T$ and large $R_T$, TEGs can only avail a limited amount of temperature-dependent available power, ranging from 0.3 nW to 3 mW, marked as $P_{IN}$ in Fig. 1. Therefore, it is essential to collect energy at the maximum power points (MPPs) to obtain the most energy.

The MPPs of TEGs are dependent on the temperature difference ($\Delta T$). Therefore, a maximum power point tracker (MPPT) is required for TEG power supplies. However, a harvester with MPPT has losses that consume power:

$$P_0 = P_{IN} - P_{LOSS}.$$  

where input power $P_{IN}$ is the power before the harvester, and output power $P_0$ represents the power available to loads as shown in Fig. 1. Hence, it is essential to maximize $P_0$, and the MPPs of $P_0$ are of utmost importance.

Several MPPT designs exist in the state of the art, with disagreement regarding which MPPs should be tracked. The majority of MPPTs stabilize the system at the MPPs of $P_{IN}$ [4]-[29], while a minority of MPPT designs stabilize the system at the MPPs of $P_0$ [30]-[33]. It remains unclear whether $P_0$ reaches its maximum when the system maximizes $P_{IN}$, and systems in [4]-[29] could potentially harvest more energy. Therefore, this paper aims to explore the theory of MPPs of $P_{IN}$ and $P_0$ and provide a discussion on the maximum input power point (MIPP) and the maximum output power point (MOPP).

This paper is organized as follows: Section II discusses $P_{IN}$ and its MPPs. Section III discusses $P_0$ and its MPPs. Section IV explores the difference between MPPs of $P_{IN}$ and $P_0$ and discusses maximum power point loss. Section V discusses the design consideration. Section VI concludes the paper.

II. MAXIMUM INPUT POWER POINT

$P_{IN}$ is the power difference between the power from $v_T$ and internal resistance loss $P_{RT}$:

$$P_{IN} = P_T - P_{RT} = v_T I_T - R_T I_T^2.$$  

To determine the current for the maximum value of $P_{IN}$, the derivative of $P_{IN}$ with respect to $I_T$ is equated to zero:

$$\frac{\partial P_{IN}}{\partial I_T} = v_T - 2R_T I_T = 0.$$  

The value of $I_T$ that satisfies this equation is denoted as $I_{T(IN)}^\prime$, which represents the MIPP:

$$I_{T(IN)}^\prime = \left(\frac{v_T}{2}\right) \left(\frac{1}{R_T}\right).$$  

At $I_{T(IN)}^\prime$, the maximum input power, $P_{IN(MIPP)}$, is obtained as:

$$R_{IN(MIPP)} = \left(\frac{v_T}{2}\right)^2 \left(\frac{1}{R_T}\right).$$  

This is the MPP as per the maximum power theorem, where the $P_{IN}$ changes across $I_T$, and maximum $P_{IN}$ is reached when $I_T$ is half $v_T$ over $R_T$, and the input voltage $v_{IN}$ is half of $v_T$.

In Fig. 2, Load represents the equivalent load seen by the TEGs, including the harvester, voltage regulator, and other
components shown in Fig. 1. The maximum value of $P_{IN}$ is
obtained when $\text{Load}$ is equal to $R_T$.

![Fig. 2. Input power of TEG harvester system.](image)

$P_{BT}$ is associated with $R_T$. As shown in Fig. 3, the higher the
$P_{BT}$ is, the lower the power generated by TEG. Hence, TEGs
with smaller values of $R_T$ are preferred when $v_T$ are the same.

![Fig. 3. Simulated $P_{IN}$ changes with different $R_T$.](image)

III. MAXIMUM OUTPUT POWER POINT

Output power $P_O$ refers to the power available to loads after $P_{IN}$
flows through the harvester, as shown in Fig. 4. Hence, the
available power for loads is $P_O$, not $P_{IN}$. TEGs typically generate
low $v_T$, which is often lower than the voltage requirements of
loads, so the harvester in Fig. 4 must provide a boosting function
switching inductors or switched capacitors since only switched
converters offer this function.

![Fig. 4. Output power of TEG harvester system.](image)

A. Losses

Switched converters generally have three types of losses. They always consume an ohmic loss $P_R$, which is proportional to the
square of $I_T$:

$$P_R = R_HI_T^2,$$

(6)

where $R_H$ is the equivalent resistance of the switched converter
used as the harvester. $P_R$ is proportional to $I_T^2$ as shown in Fig.
5. This type of loss is represented by a resistor in Fig. 4. Switched converters also have losses that are independent of $I_T$,
such as the gate drive loss $P_G$ which is produced when turning
switches on and off:

$$P_G = v_{DD}q_G = v_{DD}C_{EQ}\Delta v_C,$$

(7)

where $v_{DD}$ is the supply voltage of gate drivers, $C_{EQ}$ denotes
the equivalent gate capacitance of the switch, and $\Delta v_C$ indicates
the voltage change at the gate of the switch. To stabilize the system,
a controller is required to manage switches. The power
consumption of the controller is quiescent power $P_Q$:

$$P_Q = i_0v_{DD},$$

(8)

where $i_0$ is the total quiescent current of the controller. $P_G$ and
$P_Q$ are losses that do not depend on $I_T$ and are inherent to all
switched converters.

![Fig. 5. Simulated losses dependencies on $I_T$.](image)

In some systems, diode loss $P_D$ may occur during operation
due to a conducting diode:

$$P_D = v_DI_T$$

(9)

where $v_D$ is the voltage across the conducting diode. $P_D$ is
linearly dependent on $I_T$, which is represented by a diode in Fig.
4. The total loss $P_{LOSS}$ of a generalized harvester is:

$$P_{LOSS} = P_G + P_Q + P_D + P_R = P_HI_T^0 + v_HI_T^{-1} + R_HI_T^2$$

(10)

where $P_Q$ is the power consumption of the harvester for losses
independent of $I_T$, $v_H$ is the equivalent voltage which denotes
the losses with a constant voltage drops in the harvester.

B. Maximum Power

The power available to loads after the harvester system is
denoted as $P_O$, as shown in Fig. 4. This $P_O$ is what $P_{LOSS}$ avails
of $P_{IN}$. To determine the current for the maximum value of $P_O$,
the derivative of $P_O$ with respect to $I_T$ is equated to zero:

$$\frac{\partial P_O}{\partial I_T} = \frac{\partial (P_{IN} - P_{LOSS})}{\partial I_T} = v_T - v_H - 2(R_T + R_H)I_T \equiv 0.$$  (11)

The value of $I_T$ that satisfies this equation is denoted as $I_T^{(O)}$, which
represents the MOPP:

$$I_T^{(O)} = \left(\frac{v_T - v_H}{2(R_T + R_H)}\right).$$  (12)

At $I_T^{(O)}$, the maximum output power, $P_{O(MPP)}$, is obtained as:

$$P_{O(MPP)} = \left(\frac{v_T - v_H}{2(R_T + R_H)}\right)^2 - P_H.$$  (13)

Similar to $P_{IN}$, $P_O$ changes with $I_T$, and it reaches the maximum
when $I_T$ reaches $I_T^{(O)}$. When $I_T$ is less than $I_T^{(O)}$, $P_O$ rises as $I_T$ increases. When $I_T$ is greater than $I_T^{(O)}$, $P_O$ falls as $I_T$ increases,
as shown in Fig. 6.

![Fig. 6. Simulated $P_O$, $P_{IN}$ with rising $I_T$.](image)
IV. MAXIMUM POWER-POINT LOSS

A. MPPLOSS

$P_0$ reaches its MPP with a different current than $P_{IN}$, which means that when the system is stabilized to MIPP, $P_0$ is smaller than its maximum. This is validated by observing $P_0$ at $I_{T(IN)}'$:

$$P_0(MIPP) = P_0(MOPP) - \frac{V_H}{2} \left( \frac{1}{R_T} \right) - \frac{R_H}{R_T} \left[ \left( \frac{V_H}{V_T} \right)^2 - \left( \frac{V_T - V_H}{2} \right)^2 \left( \frac{1}{R_T + R_H} \right) \right].$$

Comparing $P_0(MOPP)$ and $P_0(MIPP)$, the first term of $P_0(MIPP)$ is $P_0(MOPP)$, and the second term of the expression is negative. Inside the third term, the first half of the expression has a smaller denominator and a larger numerator compared to the second half, which means that the third term is also negative. Consequently, $P_0(MIPP)$ is less than $P_0(MOPP)$ as shown in Fig. 6.

If the system is stabilized at $I_{T(IN)}'$, it sacrifices $P_0$. To quantify this sacrifice due to different MPPs of $P_{IN}$ and $P_0$, the maximum power points loss $MPPLOSS$ is defined as:

$$MPPLOSS = \left( \frac{\Delta P_0(MPP)}{P_0(MPPP)} \right) = \left( \frac{P_0(MOPP) - P_0(MIPP)}{P_0(MOPP)} \right) = \left( \frac{1 + \frac{R_H}{R_T}}{1 + \frac{2V_H}{V_T}} \right) \left[ \left( \frac{V_H}{V_T} \right)^2 + \left( \frac{R_H}{R_T} \right) \right] - \frac{R_H}{R_T}.$$

where $\Delta P_0(MPP)$ is the difference of $P_0$ at MIPP and MOPP respectively as shown in Fig. 6. It is the fractional opportunity loss when the system is stabilized at MIPP instead of MOPP. The higher $R_H$ and $V_H$, the higher $P_0$ is sacrificed due to MIPP.

As shown in Fig. 7, when $R_H$ is less than 10% of $R_T$ and $V_H$ is less than 5% of $V_T$, MPPLOSS is less than 2.5%, making MPPLOSS negligible. $P_0$ at MIPP and MOPP exhibit similar values, allowing flexibility in choosing either MIPP or MOPP. However, when $R_H$ is 50% of $R_T$ and $V_H$ is 23% of $V_T$, MPPLOSS reaches about 91%, which means that $P_0$ at MIPP is almost zero. Consequently, the system outputs almost no power even when operating at MIPP. In such cases, the system should operate at MOPP when MPPLOSS exceeds the accepted range, such as 91%.

B. MPP Model

$P_{IN}$ and $P_0$ are related by $P_{LOSS}$, but $I_{T(IN)}'$ is distinct from $I_{T(O)}$. Comparing $I_{T(O)}$ and $I_{T(IN)}'$, the smaller numerator and larger denominator of $I_{T(O)}$, resulting from $R_H$ and $V_H$, make $I_{T(O)}'$ consistently smaller than $I_{T(IN)}'$:

$$I_{T(O)}' < I_{T(IN)}'.$$

Thus, in the system, $P_0$ will attain the maximum power point first as $I_T$ increases, followed by $P_{IN}$ reaching its maximum power with a larger current level as shown in Fig. 6.

Fig. 8 shows an MPP model of an equivalent DC voltage source and series resistance, which simplifies the analysis of this inequality. In this model, $R_H$ and $V_H$ replace the harvester system as shown in the left circuit of Fig. 8. From the perspective of $V_O$, it has a new equivalent source voltage, which is the difference between $V_T$ and $V_H$. The new equivalent source resistance is the series combination of $R_T$ and $R_H$ as shown in the circuit on the right in Fig. 8.

C. Harvester-Charger Example

Figure 9 illustrates a boosting switched inductor, which serves as an example of a harvester-charger to validate the theory above. The circuit consists of a lithium-ion battery $V_B$ with a voltage of 2.7 V, and a voltage source $V_T$ of 200 mV with $R_T$ of 5 Q. $V_{IN}$ delivers energy to $V_B$ through inductor $L_X$, which has a series resistance $R_L$.

The ground energize switch $M_{EG}$ energizes $L_X$ from $V_{IN}$ to ground, while the output drain switch $M_{DO}$ drains $L_X$ from $V_{IN}$ to $V_B$. These switches are implemented using MOSFET. This way, inductor current $i_L$ draws $P_{IN}$ from $V_{IN}$ and outputs $P_0$ to $V_B$. The circuit, as shown in Fig. 9, is simulated under continuous conduction mode (CCM).

In CCM, $L_X$ conducts $i_L$ continuously across conduction time $t_{SW}$, as shown in the simulated $i_L$ waveform in Fig. 10 for a switching frequency of 1 MHz. When $V_{IN}$ energizes $L_X$, $i_L$ rises across energize time $t_E$, and when $V_B$ drains $L_X$, $i_L$ decreases over drain time $t_D$. To prevent shorting $V_B$ to ground, dead time $t_{DT}$ is introduced during $t_D$. During $t_{DT}$, $M_{EG}$ and $M_{DO}$ are open, allowing $V_B$ to drain $L_X$ through the dead time diode in Fig. 9.
HARVESTER DESIGN

A. Harvester Efficiency

The objective of a harvester is to obtain the highest possible PO, which requires minimizing the harvester’s losses. One common design approach for reducing losses is to maximize the harvester’s efficiency \( \eta_C \). \( \eta_C \) is the fraction of \( P_{IN} \) that reaches output as \( P_O \):

\[
\eta_C = \frac{P_O}{P_{IN}}.
\]

(17)

In the SoA, as described in [16] and [21], some designs first determine the MIPP and then attempt to maximize the \( \eta_C \) of the harvester at MIPP to maximize the \( P_O \). In other words, \( \eta_C \), \( P_{IN} \), and \( P_O \) reach the maximum at the same current level:

\[
I_{T(C)'} = I_{T(O)'} = I_{T(IN)'}.
\]

(18)

where \( I_{T(C)'} \) is the current level where maximum \( \eta_C \) is reached. However, this assertion is not always valid.

To determine the current for the maximum value of \( \eta_C \), the derivative of \( \eta_C \) with respect to \( I_T \) is equated to zero:

\[
\frac{\partial \eta_C}{\partial I_T} = \frac{P_{IN} \left( \frac{\partial P_O}{\partial I_T} \right) - P_O \left( \frac{\partial P_{IN}}{\partial I_T} \right)}{P_{IN}^2} = \frac{P_H v_T - (R_T v_{TH} + v_T R_H) I_T^2 - 2P_H R_T I_T}{\left( v_T I_T - R_T I_T^2 \right)^2} = 0.
\]

(19)

The value of \( I_T \) that satisfies this equation is denoted as \( I_{T(C)'} \), which represents the maximum efficiency point:

\[
I_{T(C)'} = \sqrt{\frac{P_H \left( v_T^2 R_T + v_T R_T v_{TH} + R_T P_H \right) - P_H R_T}{R_T v_{TH} + v_T R_H}}.
\]

(20)

By plotting \( \eta_C \), \( P_{IN} \), and \( P_O \) as shown in Fig. 11, the MIPP, MOPP, and maximum efficiency point always follow:

\[
I_{T(C)'} \leq I_{T(O)'} \leq I_{T(IN)'}.
\]

(21)

There is another way to illustrate this inequality. \( P_O \) is a \( \eta_C \) fraction of \( P_{IN} \). As the product of two peaking functions, \( \eta_C \) and \( P_{IN} \), \( P_O \) can only peak between the peaks of \( \eta_C \) and \( P_{IN} \). As discussed in section III, the ohmic loss in the harvester shifts the MOPP away from MIPP. Therefore, for harvesters with ohmic loss, it is not possible to maximize \( \eta_C \) at MIPP, and maximizing \( \eta_C \) at MIPP is not a promising design direction.

B. MPP Theory

MIPP and MOPP represent different current levels due to the ohmic losses of the harvester and are equivalent when the harvester’s \( R_H \) and \( v_{TH} \) values are zero. Since MOPP is always different from MIPP, the harvester should operate at MOPP to maximize \( P_O \). However, during the design process, there are additional factors to consider.

When \( R_H \) is less than 10% of \( R_T \) and \( v_{TH} \) is less than 5% of \( v_T \), the difference between MIPP and MOPP is less than 14%, and MPPLOSS is less than 2.5%. As a result, \( P_O(MOPP) \) is nearly identical to \( P_O(MIPP) \). If MPPLOSS is lower than the acceptable level of loss, MPPT can track either MIPP or MOPP. Designers then can choose which MPP to track based on chip area, controller power, and other factors.

The choice between MIPP and MOPP is also influenced by the characteristics of TEG. TEGs with higher \( R_T \) make it easier to restrict the harvester’s \( R_H \) to less than 10% of \( R_T \) than those with smaller \( R_T \). Therefore, TEGs with higher \( R_T \) offer more design flexibility. Likewise, TEGs with higher \( v_T \) for the same \( R_T \) provide greater flexibility for the same reason.

VI. CONCLUSIONS

This paper presents a theoretical analysis of the maximum power point (MPP) for both input power \( P_{IN} \) and output power \( P_O \) in thermoelectric-powered systems, validated by simulation. The study reveals that it’s impossible to maximize \( P_{IN} \) and \( P_O \) simultaneously when the harvester experiences ohmic loss. Instead, the system would first maximize \( P_O \) with a lower source current and then maximize \( P_{IN} \) as the source current rises. The paper also introduces the concept of maximum power point loss, which results from operating the system at different MPPs. To conceptualize the impact of harvesters on MPPs, the paper proposes an MPP model. Furthermore, the analysis of the efficiency \( \eta_C \) and MPPs indicates that maximizing \( \eta_C \) at maximum \( P_{IN} \) does not necessarily result in maximum \( P_O \). This paper demonstrates that TEGs with lower source resistance \( R_T \) offer higher \( P_O \) at the same temperature difference, while those with higher \( R_T \) provide greater design flexibility.

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