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Switched Inductor's Frequency Response

The principal aim of power supplies is to transfer input power to the output. Conditioning this power into a form that the load can receive and use is a crucial component of this directive. This is what turns *voltage regulators* into voltage sources and *battery chargers* and *light-emitting diode* (LED) *drivers* into current sources.

To suppress deviations, power supplies incorporate feedback loops that monitor and oppose output variations. This opposition is ultimately a reaction to a disturbance. So understanding how *switched inductors* (SL) respond to the adjustments that disturbances prompt is critical.

Since fluctuations decompose into frequency components, *frequency response* describes how systems react to dynamic inputs. This response is well-understood in linear systems. Nonlinear systems like the switched inductor are not so straightforward. In these cases, isolating and modeling the dynamic elements peel away the complexities of nonlinearity.

1. Two-Port Models

Two-port models are four-component networks that predict the reverse and forward response of a circuit when driven and loaded. The input and output of the model are resistive interdependent voltage or current sources. The input loads what drives the network and models *feedback* effects and the output drives the load and models *forward translations*.

The fundamental advantage of these two-port models is simplicity, because four components can emulate the effects of complex circuits. This is possible because the components model orthogonal effects. In other words, each component models what the others do not.

Since v_{IN} and i_{IN} are ohmic R_{IN} translations of one another, zeroing one eliminates the other. So nulling forward translations ultimately produce the same effect: zero v_{IN} and i_{IN}. This means that the *output resistance* R_O is also the same in all forward models.

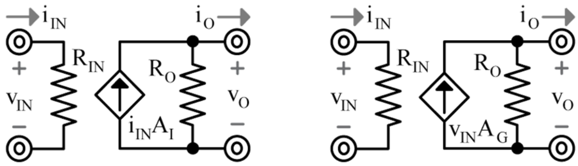

Fig. 8. Forward current-sourced models.

The only variation in forward models is the forward translation. This forward translation is a series voltage when using a voltage source and a parallel current when using a current source. And it reacts to v_{IN} or an ohmic R_{IN} translation of v_{IN}, which is what i_{IN} is.

Example 2: Extract voltage-driven current-source parameters for the impedance model when feedback effects are negligible.

Solution:

No feedback $\therefore$ $A_{ZI} \approx 0$

$R_{IN} \approx R_{ZI}$

$R_O \approx R_{ZO}$

$$A_G \equiv \left.\frac{i_O}{v_{IN}}\right|_{v_O=0} \approx \frac{i_{IN}A_{ZO}/R_{ZO}}{i_{IN}R_{ZI} + i_O A_{ZI}} \approx \frac{A_{ZO}}{R_{ZI}R_{ZO}}$$

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2. LC Primitives

2.1. Impedances

A. Capacitor

Capacitors do not consume power like resistors. Instead, they draw, cache, and supply energy. They are reactive components because they are

The gain drops with f_O in "s" when $s(R_O \| R_C)C_S$ exceeds 1, $2\pi f_O$ in "s" overcomes $2\pi p_C$, or more simply, f_O surpasses p_C, as already stated. A_V's magnitude $|A_V|$ is the ratio of the *root–sum of squares* (RSS) of the real and imaginary components in A_{V0} and $1 + s(R_O \| R_C)C_S$ or $1 + s/2\pi p_C$:

$$|A_V| = \frac{\sqrt{{A_{V0}}^2 + 0^2}}{\sqrt{1^2 + (f_O/p_C)^2}} = \frac{A_{V0}}{\sqrt{1^2 + (f_O/p_C)^2}}. \tag{7}$$

So $|A_V|$ is nearly A_{V0} a decade below p_C, $\sqrt{2}$ or 3 dB lower at p_C, and almost 10× or 20 dB lower a decade past p_C.

Phase: Since C_S requires time to charge (raise v_O), C_S delays v_{IN}-to-v_O translations. The delay between v_{IN} and v_O sinusoids when $1/sC_S$ swamps $R_O \| R_C$ effects is 90° of the 360° cycle. This lagging (negative) delay halves when $1/sC_S$ matches $R_O \| R_C$ and fades when C_S opens. So $\angle A_V$ is near 0° a decade below p_C, –45° at p_C, and almost –90° a decade past p_C:

$$\angle A_V = -\tan^{-1}\left(\frac{f_O}{p_C}\right). \tag{8}$$

Transitional frequencies that prompt gain and phase to drop 20 dB per decade and up to 90° this way are *poles*.

B. Current-Limit Resistor

The shunting effects of capacitors disappear when they short. And they effectively short when resistors limit their current. Consider C_S and *current-limit resistor* R_I in Fig. 10, for example.

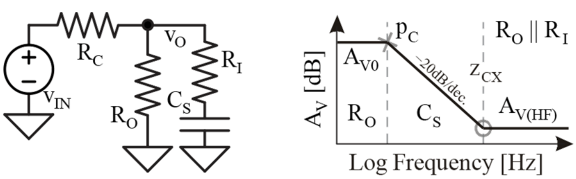

Fig. 10. Current-limited shunt capacitor.

Gain: R_I adds to the resistance that R_O and R_C into v_{IN} present. So C_S shunts R_I and R_O with R_C past the p_C that their RC frequency sets:

C_B feeds a current that climbs with f_O. Since C_O's current also climbs with f_O, i_B effectively replenishes the i_C that C_O sinks. This means that z_C cancels the shunting effects of C_O. In other words, z_C recovers the phase that a pole loses. So $\angle A_G$ is nearly 0° a decade below z_C, +45° at z_C, and almost +90° a decade past z_C:

$$\angle A_G = +\tan^{-1}\left(\frac{f_O}{z_C}\right). \tag{53}$$

B. Out-of-Phase Capacitor

In Fig. 21, the amplifier is inverting because a rise in v_{IN} increases an i_{GO} that reduces v_O. Like in Fig. 19, i_B overcomes i_{G0} when $v_{IN}sC_B$ surpasses $v_{IN}A_{G0}$. So the gain A_{G-} to i_G increases with i_B past $A_{G0}/2\pi C_B$.

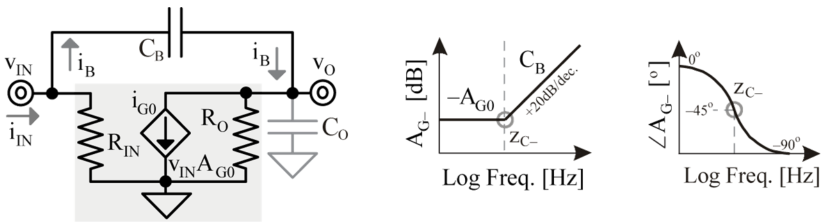

Fig. 21. Out-of-phase bypass capacitor across amplifier.

The effects of i_B and i_{G0} on v_O in Fig. 21, however, oppose because C_B feeds i_B while A_{G0} sinks i_{G0}. This means that i_B and i_{G0} are out of phase, and the resulting zero z_{C-} inverts 180° while also recovering 90°. So $\angle A_{G-}$ is near 0° a decade below z_{C-}, –90° + 45° or –45° at z_{C-}, and almost –180° + 90° or –90° a decade above z_{C-}:

$$\angle A_{G-} = -\tan^{-1}\left(\frac{f_O}{z_{C-}}\right). \tag{54}$$

The overall gain to i_G's i_B and i_{G0} is the ohmic and transconductance v_{IN} translations that $1/sC_B$ and A_{G0} set when v_O is zero:

$$A_{G-} \equiv \left.\frac{i_G}{v_{IN}}\right|_{v_O=0} \equiv \left.\frac{i_B - i_{G0}}{v_{IN}}\right|_{v_O=0}$$

This $A_{Z(LC)}$ is above Z_L's f_{LC} projection $Z_{L(LC)}$ when A_Z rises and falls with Z_L and Z_C. In other words, A_Z peaks when L_X and C_O interact at f_{LC}.

For this, R_P should not voltage-limit L_X (past f_{LP}) below f_{LC}. This is like saying C_O shunts R_P (past f_{CP}) below f_{LC}. This is because R_P is greater than Z_L at f_{LC} when Z_L cannot surpass R_P. This way, A_Z follows Z_L to f_{LC}.

A_Z peaks above $Z_{L(LC)}$ when f_{CP} precedes f_{LC} and f_{LP} surpasses f_{LC}. In this light, f_{LP}/f_{LC} and f_{LC}/f_{CP} in Q_{LC} indicate the *quality* and *magnitude* of the *peak*. Peaking occurs when R_P raises this Q_{LC} above 1:

$$Q_{LC} \equiv \frac{f_{LP}}{f_{LC}} = \frac{f_{LC}}{f_{CP}} = R_P\sqrt{\frac{C_O}{L_X}}\ . \tag{61}$$

But if Z_L surpasses R_P before f_{LC}, f_{LP} sets a p_L that precedes f_{LC}. Since R_P is less than Z_L at f_{LC} this way, C_O shunts R_P past a p_C that f_{CP} sets above f_{LC}. p_L and p_C split away from f_{LC} this way because R_P keeps L_X and C_O from interacting at f_{LC}.

C. Resistive Effects

In practice, electrical components incorporate series resistance. L_X and C_O in Fig. 24, for example, includes the *inductor* and *capacitor resistances* R_L and R_C in Fig. 25. Circuits that connect to v_O also add *load resistance* R_{LD}.

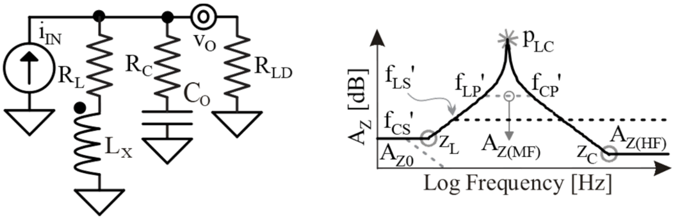

Fig. 25. Loaded current-sourced LC with parasitic resistances.

<u>Peaked</u>: R_L alters A_Z's low-f_O gain. Since L_X shorts and C_O opens at low f_O, the low-f_O gain to v_O is an ohmic R_L and R_{LD} translation of i_{IN}:

$$A_{Z0} \approx R_L \,||\, R_{LD}\,. \tag{62}$$

A_Z climbs past the z_L that f_L sets when sL_X overcomes R_L:

resulting *inductor voltage* v_L pulses and swings the *inductor current* i_L across amp-level ramps. These variations in v_L and i_L are large signal.

But when conditions settle, the power supply reaches a steady state that pulses v_L and ripples i_L to peaks and about averages that do not vary much over time. So cycles repeat and variations between cycles fade. This is the static periodic steady state of the switched inductor.

When operating conditions vary, the controller uses the error it senses in the output to adjust the switching cycle by a small amount. This small variation in amplitude, duty cycle, or frequency is the dynamic manifestation of the feedback command. So cycle-to-cycle variations in v_L and i_L reflect small-signal translations across the switched inductor.

5.2. Small-Signal Model

The switched inductor is a network of switches that connects L_X in Fig. 31 to the *input* v_{IN}, *output* v_O, and ground. The digital input that adjusts the connectivity of the network is the *energize command* d_E'. d_E' feeds logic that configures the switches so L_X energizes when d_E' is high.

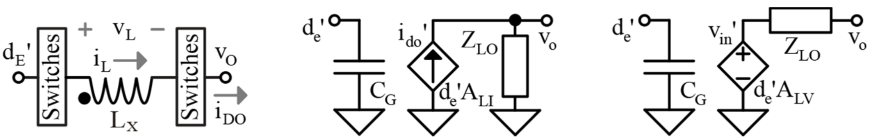

Fig. 31. Small-signal model of the switched inductor.

d_E' is the fraction of the *switching period* t_{SW} that energizes L_X. This d_E' in modern power supplies connects to the gates of *complementary metal–oxide–semiconductor* (CMOS) transistors. These gates are capacitive and largely insensitive to the behavior of the network. So d_E' connects to gate capacitance C_G and the small-signal model that incorporates this C_G excludes a feedback translation.

Variations in d_E' ultimately alter the *output duty-cycled current* i_{DO} that the network outputs. Z_{LO} is the output impedance of the network in the

B. Discontinuous Conduction

L_X in *discontinuous-conduction mode* (DCM) conducts a fraction of t_{SW}. This is because L_X energizes and drains before t_{SW} in Fig. 34 ends. So i_L climbs with v_E, falls with v_D, and flattens until another t_{SW} begins.

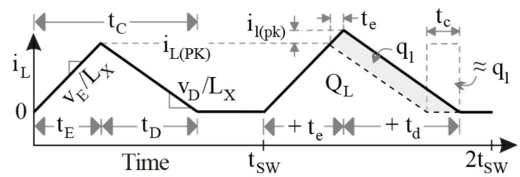

Fig. 34. Small-signal inductor-current variation in DCM.

Several observations are worth noting. First, d_E and d_D are t_E and t_D fractions of a t_C that is shorter than t_{SW}. So d_E is not the t_E/t_{SW} that the input d_E' commands.

Second, L_X energizes and depletes every cycle. So L_X delivers all the charge it collects. This means that i_{do}' does not lose the i_e that inverts and alters i_{do}' in CCM. In other words, A_{LI} and A_{LV} in DCM exclude z_{DO}.

Third, extending t_E in DCM extends t_D when t_{SW} is constant. This is because adding t_e raises $i_{L(PK)}$, and with it, the $0.5L_Xi_{L(PK)}{}^2$ energy that L_X collects. So L_X requires more t_D to drain.

In fact, t_D and t_C scale proportionately with t_E within t_{SW} because the v_E and v_D that project i_L are static. So the fraction of v_E that d_e applies to L_X cancels the fraction of v_D that d_d applies. This means that the small-signal voltage v_l across L_X is zero:

$$\frac{v_l}{Z_L} = \frac{V_E d_e - V_D d_d}{Z_L} = 0\,. \tag{93}$$

Losing sensitivity to f_O removes inductive effects from A_{LI}, A_{LV}, and Z_{LO}.

Since v_E and v_D projections are static, variations in t_E induce variations in t_C that track T_E and T_C and the D_E that T_E and T_C set:

Since poles also delay and suppress signals, a *switching pole* p_{SW} can model this behavior. Although the delay and suppression are not always linear or consistent, p_{SW} is a useful, albeit imperfect way of indicating the switcher filters higher-f_O signals. Note this p_{SW} affects the d_e' that feeds and propagates to i_l', d_{do} i_{do}', and v_{in}'.

5.3. Power Stage

Switched inductors deliver v_{IN} energy with i_{DO}. Engineers add C_O in Fig. 36 to voltage regulators and LED drivers to supply i_O when d_{DO} disconnects L_X from v_O. In the case of chargers, C_O is the effective capacitance of the battery, which is usually very high. R_L and R_C are the parasitic series resistances into L_X and C_O. i_{LD} is the static part of the load and R_{LD} is the part that responds to small v_O variations v_o.

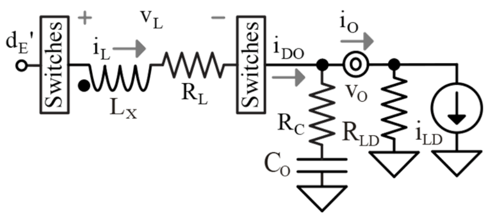

Fig. 36. Power stage.

The small-signal gain to v_O is critical in voltage regulators because the feedback loop regulates this v_O. In LED drivers, i_O is more important because the brightness and spectrum of the emitted light depend on the i_O the LEDs receive. i_{DO} is critical in chargers because i_{DO} is what feeds the battery. i_L is also important when systems control the i_L that L_X conducts. This is why small-signal d_E' translations to v_O, i_O, i_{DO}, and i_L are relevant.

A. Continuous Conduction

The switched inductor in continuous conduction is a voltage-sourced inductor. So when R_L is low, the switcher in Fig. 36 sets a small-signal voltage v_{in}' in Fig. 37 that drives L_X's L_{LO} into C_O and R_{LD}. i_{LD} is absent because i_{LD} does not respond to small signals.

$$I_{L(PK)} = \left(\frac{V_E}{L_X}\right) D_E T_C = \left(\frac{2}{10\mu}\right)(50\%)(570n) = 57 \text{ mA}$$

$$R_{LO} = 2\left(\frac{L_X}{D_{DO}^{\ 2}}\right)\left(\frac{T_{SW}}{T_C^{\ 2}}\right) = 2\left(\frac{10\mu}{50\%^2}\right)\left(\frac{1\mu}{570n^2}\right) = 250 \ \Omega$$

$$A_{LI} \approx \frac{I_{L(PK)}}{D_E} = \frac{57m}{50\%} = 110 \text{ mA/V}$$

$A_{VO0} = A_{LI}(R_{LO} \parallel R_{LD}) = (110m)(250 \parallel 500) = 18$ V/V

$$A_{DO0} = \frac{A_{VO0}}{R_{LD}} = \frac{18}{500} = 36 \text{ mA/V}$$

$$z_{CP} = \frac{1}{2\pi\left(R_C + R_{LD}\right)C_O} = \frac{1}{2\pi(10m + 500)(5\mu)} = 64 \text{ Hz}$$

$$p_C = \frac{1}{2\pi\left[R_C + \left(R_{LO} \parallel R_{LD}\right)\right]C_O}$$

$$= \frac{1}{2\pi[10m + (250 \parallel 500)](5\mu)} = 190 \text{ Hz}$$

$p_{SW} \approx f_{SW} = 1$ MHz from Example 17

$z_C = 3.2$ MHz from Example 9

Note: z_{CP} is in A_{DO} because A_{DO} is an ohmic Z_C translation of A_{VO}.

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6. Summary

Two-port models are two- to four-component networks that can model the behavior of almost any circuit. Their inputs and outputs are interdependent sources with impedances. These work because sources model what impedances do not. In other words, sources are the voltages or currents that result when the effects of impedances are absent and impedances are the ohmic translations that result when disabling sources. This way, input–output combinations can model feedback and forward translations.