Abstract—Inductively coupled power receivers for embedded microsensors are often tiny and distant from their transmitting sources. To sustain microsensors as long as possible, the power receiver should draw power whenever possible and output the highest power possible. Of reported state-of-the-art technologies, switched resonant half-bridges require fewer components, are less breakdown-limited, and output as much or more power than the others. This paper derives and shows with measurements the highest possible maximum power point (MPP) for switched resonant half-bridges. The theory predicts the optimal time, duration, and frequency of the energy transfers that charge the battery. Measurements of a 0.18-µm CMOS power receiver demonstrate that the receiver outputs more than 98.7% of the actual MPP at the predicted settings when the coupling factor between the transmitting and receiving coils is 0.15%–1.14%.

Index Terms—Inductively coupled power receiver, maximum power-point theory, switched resonant half-bridge, wireless power transfer.

I. POWERING EMBEDDED MICROSENSORS

Structurally embedded microsensors and biomedical implants can sense, process, and transmit data that save energy, money, and lives [1]-[5]. Powering these microsensors is challenging because the tiny batteries that they incorporate cannot sustain their operation for long, so they need to be replenished often. Harvesting ambient energy can help, but energy sources like light or motion are rarely available in an embedded environment. Temperature gradients can also generate power, except the temperature difference across tiny devices is normally too low to generate practical power levels [6]. Often, the only option left is to transfer power wirelessly via a pair of inductively coupled coils.

To transfer power wirelessly, the transmitter’s source $v_S$ in Fig. 1 injects energy into $L_TC_T$. The oscillating current in $L_T$ generates a changing magnetic field. The receiving coil $L_R$ captures the magnetic flux that $L_T$ emits, and induces an electromotive force (EMF) voltage $v_E$. To draw power, the power receiver conditions and harnesses the current from $v_E$. The receiver charges the battery $v_B$ with this power so that a power supply can feed the sensor, amplifier, analog-to-digital converter (ADC), digital-signal processor (DSP), and power amplifier (PA) that comprise the microsystem.

To sustain the microsystem as long as possible, the power receiver should draw power whenever possible and output the highest power possible. Most embedded microsensors are far away (radially distant) from their transmitting sources [7]-[10], so the coupling factor $k_C$ between the coils is very low. As a result, $v_E$ is often tens of millivolts [11]. The only way to draw more power from such a low $v_E$ is to raise the inductor current $i_L$ [12]. Applying an alternating high voltage over $L_R$ can boost $i_L$. Except, $L_R$'s resistance limits this $i_L$ [13]. The circuit’s breakdown voltage, which is often only a few volts for deep submicron CMOS technologies, also limits the maximum voltage that can be applied. So the output power is both loss- and breakdown-limited [12].

This paper identifies the highest power-producing receiver in the state of the art and presents and validates (with simulations and measurements) theory that produces the highest maximum power point (MPP) possible. For this, Sections II first compares the power performance and limitations of the state of the art. Section III analyzes the highest power-producing receiver. Section IV then predicts the settings that produce the highest MPP possible. Section V validates this theory with simulations and measurements and Section VI summarizes and draws overarching conclusions.

II. POWER RECEIVERS

A. Resonant Bridge

The resonant bridge [14] in Fig. 2 parallels a resonant capacitor $C_R$ to $L_R$. Since $L_R$ and $C_R$ resonate at $v_E$’s operating frequency $f_O$, $v_E$ constantly sources power, so the oscillation grows. As $v_C$ grows above $v_{REC}$, diodes $D_O^+$ and $D_G^+$ turn on and steer $i_L$ into $C_{REC}$. Similarly, as $v_C$ grows below $–v_{REC}$, $D_O^-$ and $D_G^-$ turn on and steer $i_L$ to charge up $C_{REC}$. $v_{REC}$ therefore limits $v_C$’s peak to $v_{REC}$.
In practice, $D_{G}^{-}$, $D_{G}^{+}$, $D_{G}^{-}$, and $D_{G}^{+}$ are usually comparator-based MOSFET switches because they drop millivolts [15], [16]. Ground or output diodes can be replaced with a pair of gate cross-coupled transistors to save area and power [17]. Such power receivers normally operate at megahertz. [1], [14], [16]-[18], for example, operate at 6.78 or 13.56 MHz.

The resonant bridge draws the highest power from $L_R$ when it draws as much power as the resistance $R_{ESR}$ in the LC tank loses [12]:

$$P_{L(\text{MAX})} = \frac{(0.5v_{E(PK)})^2}{2R_{ESR}}.$$  \hspace{1cm} (1)

At this point, $R_{ESR}$ shares 0.5$v_E$ [19]. Since the reactance of $C_R$ is $Q_R$ times higher than $R_{ESR}$, the optimal capacitor voltage $v_{C(OPT)}$ is [12]:

$$v_{C(OPT)} = Q_Rv_{ESR(OPT)} = 0.5Q_Rv_{E(PK)},$$  \hspace{1cm} (2)

where $Q_R$ is the quality factor of the LC tank.

To operate at the maximum power point (MPP), the resonant bridge requires another power stage: an MPP stage that regulates $v_{REC}$ near $v_{C(OPT)}$. This additional stage adds losses and more components to the system. Since all four diodes see $v_C$, $v_{C(PK)}$ is limited to the breakdown voltage $V_{BD}$.

When $v_E$ is much lower than $v_{REC}$, $v_E$'s waveform is triangular [12]. Although the switched bridges in [8] and [9] work at 125 kHz, the same operating principle applies at megahertz. Similarly, the switched bridge draws the most power when $v_{REC}$ is optimal [12]:

$$P_{L(\text{MAX})}\big|_{v_{REC(OPT)}} = \frac{12v_{E(PK)}^2}{\pi^2R_{ESR}}.$$  \hspace{1cm} (3)

For MPP, the switched bridge also needs an MPP stage to regulate $v_{REC}$. The circuit's $V_{BD}$ also limits the maximum $v_{REC}$ because all four switches see $v_{REC}$. To operate, the circuit needs to synchronize the switches to $v_E$. In [9], the switched bridge synchronizes the switches by interrupting the operation and sensing the open-circuit $v_E$ every 11 cycles. The circuit loses 13% of its power to this interruption [9].

C. Switched Resonant Half-Bridge

The switched resonant half-bridge [10], [11], [20], [21] uses $C_R$ to keep $v_E$ and $i_L$ in phase. A pair of switches, $S_G$ (or $S_C$) and $S_O$ in Fig. 4 control the energy transfer. As $S_O$ or $S_C$ closes, the LC tank receives power from $v_E$. So $v_{C(PK)}$ grows from cycle to cycle. As $S_O$ closes and $S_G$ (or $S_C$) opens, the LC tank partially transfers its energy to $v_B$. The switched resonant half-bridge can be categorized into two types: the series type and the parallel type. For the series type in Fig. 4a, $v_E$ drains energy from $L_R$ in series [11]. For the parallel type in Fig. 4b, $S_C$ opens and disconnects $C_R$, so $v_E$ drains energy from $L_R$ alone [20]. The switched resonant half-bridge can operate from 50 kHz to 6.78 MHz [10], [11], [20], [21].

![Switched Resonant Bridge](image)

Fig. 4. Switched resonant bridges: (a) series and (b) parallel capacitors.

When equally coupled, all three types of power receivers can output about the same power (from Eqs. 1 and 3). The switched resonant half-bridge, however, does not need the additional MPP stage that the other two require. So the switched resonant half-bridge consumes less power, and as a result, outputs more power. Between the series and parallel options, the series circuit is less breakdown-limited, so it can ultimately output more power.

III. HIGHEST POWER-GENERATING RECEIVER

Figure 5 models the series-switched resonant receiver. The transmitter couples an open-circuit voltage $v_E$ in $L_R$. $R_C$ is the reflected resistance from the transmitter [22]. As the receiver draws power, it loads the transmitter, so it lowers the current in the transmitting coil as well as $v_E$ in $L_R$. The voltage dropped across $R_C$ models this loading effect. The power receiver is modeled as a series load with a terminal voltage $v_R$. When $S_G$ in Fig. 4a closes, the ground switch shorts $L_R$ and $C_R$, so $v_R$ in Fig. 5 is zero. When $S_O$ closes, $L_R$ is connected to $v_B$, so $v_R$ equals $v_B$. As the circuit switches periodically, $v_R$'s waveform is a pulse train.

![Series-Switched Resonant Power Receiver](image)

Fig. 5. Series-switched resonant power receiver.

A power stage that can adjust its energy transfer frequency, duration, and phase is proposed in [11]. This paper presents a theory that predicts the optimal (maximum power-point) settings for frequency, duration, and phase, which [11] did not include. The phase offset $t_{OS}$ is the phase difference between $v_R$ and $v_E$. When $t_{OS}$ is zero, $v_E$ peaks halfway across $v_R$'s...
pulse. For example, in Fig. 6, as the center of \( v_R \)’s pulse lags \( v_E \)’s peak by 10 ns, \( t_{OS} \) is 10 ns. The circuit can also adjust the energy-transfer frequency \( f_X \) by setting the number of cycles \( N_S \) between two consecutive transfers. In Fig. 6, as the circuit transfers energy to \( v_B \) every 5 cycles, \( t_X \) is 5 ns and \( f_X \) is \( f_O/5 \). Finally, the circuit can adjust the duration \( t_{ON} \) of the transfer: the time that the LC tank connects to \( v_B \). In Fig. 6, \( v_B \) drains \( L_R C_R \) for 20 ns, so \( t_{ON} \) is 20 ns.

The overall nonlinear loss \( P_{NL} \) is the average over \( N_S \) cycles:

\[
P_{NL} = \frac{1}{N_S} \sum_{i=1}^{N_S} (P_{NL}(i) - P_{NL}(i-1)).
\]

where \( C_{EQ} \) is the total equivalent capacitance that charges and discharges in one switching cycle. \( k_{SW} \) is the soft-switching factor. Without \( P_{DT} \), \( M_O \) hard-switches: burns power to charge \( v_S \)’s parasitic capacitance \( C_{EQ} \), so \( k_{SW} = 1 \). With \( P_{DT} \), \( L_X \)’s current charges \( C_{EQ} \) before \( M_O \) closes, so \( M_O \) partially soft-switches: closes with lower than \( v_B \) voltage drop. \( L_X \)’s current also charges \( C_{EQ} \) above \( v_B \) before \( M_G \) closes, so \( M_G \) switches with higher charge loss. As a result, \( k_{SW} \) can be higher or lower than 1. The analysis assumes \( k_{SW} = 1 \). The accuracy of this assumption will be verified by simulations and measurements later. Output power \( P_O \) is the \( P_{L(MAX)} \) that nonlinear and charge losses \( P_{NL} \) and \( P_{C} \) avail:

\[
P_O = P_{L(MAX)} - P_{NL} - P_{C}.
\]

The theory below finds MPP settings \( t_{OS(MPP)} \) \( t_{ON(MPP)} \), and \( f_X(MPP) \) in two steps. The theory first finds the optimal \( t_{OS} \) and \( t_{ON} \) settings that maximize \( P_O \) at a given \( f_X \). The optimal settings derived \( t_{OS(MPP)} \) \( t_{ON(MPP)} \) are "local" because they vary with \( f_X \). As will be shown, \( t_{OS(MPP)} \) \( t_{ON(MPP)} \) are the \( t_{OS} \) and \( t_{ON} \) that ensure \( v_R \)’s 1st harmonic is 0.5\( v_E \). The optimal "global" setting derived for \( f_X \) in step two \( f_X(MPP) \) minimizes the losses at this point: when \( v_R \)’s 1st harmonic is 0.5\( v_E \). With this, the optimal global setting for \( t_{OS} \) \( t_{ON} \) \( f_X \) is now known: from \( t_{ON(MPP)} \) in step one and \( f_X(MPP) \) in step two.

A. Available Power

The receiver draws the highest power when the receiver load matches the source resistance \( R_C + R_L \) [19]:

\[
P_{L(MAX)} = \frac{(0.5v_E)^2}{2(R_C + R_L)}. \tag{4}
\]

In practice, \( S_G \) and \( S_O \) also add resistance to the loop, lowering the maximum available power from the coupled source:

\[
P_{L(MAX)}' = \frac{(0.5v_E)^2}{2(R_C + R_L + R_{ESR})} = \frac{(0.5v_E)^2}{2R_{ESR}} \tag{5}
\]

Note the reflected \( R_C \) in Eq. (5) varies with \( k_C \). At lower \( k_C \), the transmitter reflects less \( R_C \) on the receiver. As a result, \( R_{ESR} \) and \( P_{L(MAX)}' \) also vary with \( k_C \).

B. Output Power

Drawing the maximum available power \( P_{L(MAX)}' \) presupposes \( v_C(OPT) \) every cycle. However, as Fig. 6 illustrates, \( v_C(OPT) \) grows from cycle to cycle between energy transfers. \( v_C(OPT) \) therefore deviates from \( v_C(OPT)_0 \), so actual drawn power \( P_l' \) is lower than \( P_{L(MAX)}' \). As depicted in Fig. 7, the difference between the drawn power \( P_{L(i)}' \) at the \( i \)-th cycle and \( P_{L(MAX)}' \) is defined as the nonlinear loss \( P_{NL(i)}' \), since the loss is due to the nonlinearity in the operation. As \( P_l' \) peaks parabolically at \( v_C(OPT) \) [13], \( P_{NL(i)}' \) grows quadratically with \( v_C(OPT)_i \) deviations:

\[
P_{NL(i)}' = P_{L(MAX)}' \left( \left( \frac{v_C(OPT)_i}{v_C(OPT)} \right)^2 - 1 \right).
\]

The overall nonlinear loss \( P_{NL} \) is the average over \( N_S \) cycles:

\[
P_{NL} = \frac{1}{N_S} \sum_{i=1}^{N_S} (P_{NL}(i) - P_{NL}(i-1)).
\]

The circuit also loses charge power as parasitic capacitances charge and discharge. This charge loss \( P_{C} \) is proportional to \( f_X \):

\[
P_{C} = v_{DD}(Q_{F}) = \frac{v_{DD}^2}{2}k_{SW}C_{EQ}f_X,
\]

where \( C_{EQ} \) is the total equivalent capacitance that charges and discharges in one switching cycle. \( k_{SW} \) is the soft-switching factor. Without \( P_{DT} \), \( M_O \) hard-switches: burns power to charge \( v_S \)’s parasitic capacitance \( C_{EQ} \), so \( k_{SW} = 1 \). With \( P_{DT} \), \( L_X \)’s current charges \( C_{EQ} \) before \( M_O \) closes, so \( M_O \) partially soft-switches: closes with lower than \( v_B \) voltage drop. \( L_X \)’s current also charges \( C_{EQ} \) above \( v_B \) before \( M_G \) closes, so \( M_G \) switches with higher charge loss. As a result, \( k_{SW} \) can be higher or lower than 1. The analysis assumes \( k_{SW} = 1 \). The accuracy of this assumption will be verified by simulations and measurements later. Output power \( P_O \) is the \( P_{L(MAX)} \) that nonlinear and charge losses \( P_{NL} \) and \( P_{C} \) avail:

\[
P_O = P_{L(MAX)} - P_{NL} - P_{C}.
\]

IV. MAXIMUM POWER

The theory below finds MPP settings \( t_{OS(MPP)} \) \( t_{ON(MPP)} \), and \( f_X(MPP) \) in two steps. The theory first finds the optimal \( t_{OS} \) and \( t_{ON} \) settings that maximize \( P_O \) at a given \( f_X \). The optimal settings derived \( t_{OS(MPP)} \) \( t_{ON(MPP)} \) are "local" because they vary with \( f_X \). As will be shown, \( t_{OS(MPP)} \) \( t_{ON(MPP)} \) are the \( t_{OS} \) and \( t_{ON} \) that ensure \( v_R \)’s 1st harmonic is 0.5\( v_E \). The optimal "global" setting derived for \( f_X \) in step two \( f_X(MPP) \) minimizes the losses at this point: when \( v_R \)’s 1st harmonic is 0.5\( v_E \). With this, the optimal global setting for \( t_{OS} \) \( t_{ON} \) \( f_X \) is now known: from \( t_{ON(MPP)} \) in step one and \( f_X(MPP) \) in step two.

A. Optimal Receiver Voltage

Ideally, the power receiver in Fig. 5 outputs the highest power when the receiver load matches \( R_{ESR} \), so \( v_R \) is 0.5\( v_E \). In reality, \( v_R \) is not a sinusoidal wave but a pulse train with a frequency \( f_X \) that is \( f_O/N_S \). Fourier series decomposes \( v_R \) as the sum of sinusoidal waveforms at \( f_X \) and its harmonics. The \( N_S \)-th harmonic of \( v_R \) is at \( f_O \). Since the series \( L_R C_R \) only passes current at \( f_O \), \( v_R \)’s harmonic at \( f_O \) dominates the current and the conduction loss. Therefore, analogous to the linear maximum power transfer theory [19], the proposed theory asserts that at given \( f_X \) the receiver draws maximum power when \( v_R \)’s harmonic at \( f_O \) matches 0.5\( v_E \) in both amplitude and phase:

\[
V_{R(PK)}(f_O) = 0.5V_{E(PK)} \quad \angle V_{R(PK)}(f_O) = \angle V_{E}.
\]

Expanding \( v_R \)’s Fourier series at \( f_O \) relates \( t_{ON(MPP)} \) and \( f_O \):

\[
V_{R(PK)}(f_O) = 2 \cdot \frac{1}{\pi} \int_{-\pi}^{\pi} \left| v_R(t) \right| dt. \tag{6}
\]

Solving (10) gives the local \( t_{ON(MPP)} \) at given \( f_X \):

\[
t_{ON(MPP)} = \left( \frac{t_{ON}}{\pi} \right) \sin^{-1} \left( \frac{\pi}{4} \frac{V_{R(PK)}}{V_{B}} \right). \tag{12}
\]
From (11), since \( v_R \)’s harmonic at \( f_0 \) and \( v_E \) are in phase, the center of the pulse aligns with \( v_E \)’s peak. Therefore, the phase offset at the local and the global MPP is zero:

\[
t_{OS(MPP)}' = t_{OS(MPP)} = 0. \quad (13)
\]

Fig. 8 plots the simulated and measured \( P_O \) across \( t_{OS} \) when \( f_X = 850 \text{ kHz} \) and \( t_{ON} = 25 \text{ ns} \). In both simulation and measurement, the circuit outputs the highest power when \( t_{OS} = 0 \). \( P_O \) drops as \( t_{OS} \) deviates from zero in either direction. Fig. 9 plots the calculated, simulated, and measured \( V_{R(PK)} \) at MPP when \( k_C \) is 0.07–1.13%. At \( k_C > 0.15\% \), both simulated and measured \( V_{R(PK)} \) deviates up to 21% from 0.5\( V_{E(PK)} \), due to the finite resolutions for \( t_{ON} \) and \( f_X \) (5 ns and 53 kHz, respectively).

Fig. 10 plots the simulated and measured \( P_O \) across \( t_{ON} \) when \( f_X = 0.125f_0 \). The simulated \( P_O \) peaks at \( t_{ON} = 23 \text{ ns} \), deviating 1 ns from the theory’s predicted 24 ns. The measured \( P_O \) peaks at \( t_{ON} = 25 \text{ ns} \) with 5 ns resolution. Simulated \( k_{SW} \) ranges from 0.86% to 1.14%. To evaluate the accuracy of the predicted \( t_{ON(MPP)}' \) in (12), Fig. 11 compares the calculated, simulated, and measured \( t_{ON(MPP)}' \) across \( f_X \) with 118 mV of \( V_{E(PK)} \) and 6.2 \( \Omega \) of \( R_{ESR} \). While the simulated \( t_{ON(MPP)}' \) closely matches the theory’ prediction, the measured \( t_{ON(MPP)}' \) has a resolution of 5 ns and reflects the trend of prediction.

**B. Optimal Transfer Frequency**

The next step for MPP is to find the optimal \( f_X \) that maximizes the global \( P_O \) with the corresponding \( t_{OS(MPP)}' \) and \( t_{ON(MPP)}' \) found in the previous step. For this, as discussed in Section III, \( f_X \) needs to minimize the total losses \( P_{NL} + P_C \). Between energy transfers, the LC tank collects across cycles the power \( L_R \) sources. The difference between consecutive cycles \( i - 1 \) and \( i \)’s peak \( C_R \) energy, therefore, reflects energy the \( L_R \) sources across cycle \( i \) [13]:

\[
P_{L(i)}' = P_{L(MAX)}' - P_{NL(i)}' = P_{L(MAX)}' \left[ 1 - \left( \frac{V_{C(i)} - V_{C(OPT)}}{V_{C(OPT)}} \right)^2 \right]. \quad (14)
\]

Replacing \( P_{L(i)}' \) in (14) with (15), \( V_{C(PK)} \) can be calculated iteratively from cycle to cycle:

\[
V_{C(i)} = \pi V_{E(PK)} + V_{C(i-1)} \left( 1 - \frac{\pi}{Q_R} \right). \quad (16)
\]

Assume \( V_{C(PK)} \) is closest to \( V_{C(OPT)} \) at the m-th cycle, using (16), \( V_{C(PK)} \) at i-th cycle can be calculated as:

\[
V_{C(i)} = 2V_{C(OPT)} - V_{C(OPT)} \left( 1 - \frac{\pi}{Q_R} \right). \quad (17)
\]

Replacing \( V_{C(i)} \) in (6) with (17) and assuming \( P_O \) is locally maximized, \( P_{NL} \) at the local MPP can be expressed as:

\[
P_{NL(MPP)}' = P_{L(MAX)}' \left[ 1 - \left( \frac{4a}{1-a^2} \right) \left( \frac{1-a^2}{1+a^2} \right) \left( \frac{f_X}{f_0} \right) \right]. \quad (18)
\]

where \( a = 1 - \frac{\pi}{Q_R} \). Since \( P_C \) is not a function of \( t_{OS} \) or \( t_{ON} \), \( P_{C(OPT)}' = P_C \) at \( f_X \) as Fig. 12 shows, the theory predicts \( P_{NL} \), \( P_C \), and thus \( P_O \) at the local MPP. For global MPP, \( f_{X(MPP)} \) needs to minimize the total losses \( P_{NL(MPP)}' + P_C' \):

\[
\frac{\delta P_{NL(MPP)}'}{\delta f_X} = - \left( \frac{\delta P_{NL(MPP)}'}{\delta f_X} - \frac{\delta P_C'}{\delta f_X} \right) f_{X(MPP)} = 0. \quad (19)
\]

With the \( P_{NL(MPP)}' \) obtained in (18) and \( P_C \) obtained in (8), solving (19) gives \( f_{X(MPP)}' \):
\[
\begin{align*}
\text{where } P_{\text{C}}(f) \text{ is the charge loss when } f_X = f_O. \text{ The global } \\
\text{optimal } t_{\text{ON(MPP)}} \text{ is, therefore, } t_{\text{ON(MPP)}}' \text{ at } f_{X(MPP)}: \\
&\quad \text{t}_{\text{ON(MPP)}}' = \frac{t_{\text{ON(MPP)}}}{f_{X(MPP)}}.
\end{align*}
\]

Detailed derivations of (16), (17), (18), and (20) are included in the Appendix. Fig. 12 compares the simulated losses with the theory’s prediction in (8) and (18). The theory accurately predicts the simulated \( P_{\text{NL}} \) and \( P_{\text{C}} \) with less than 5% error when \( f_X \) is 420 kHz–3 MHz.

C. Maximum Output Power

With the losses (\( P_{\text{NL}} \) and \( P_{\text{C}} \)) and MPP settings (\( t_{\text{OS(MPP)}}, t_{\text{ON(MPP)}}, f_{X(MPP)} \)) obtained in the previous two subsections, the output power is \( P_{\text{MPP}} \) minus the losses at the MPP settings:

\[
P_{\text{MPP}} = P_{\text{L(MAX)}} - P_{\text{NL(MPP)}} - P_{\text{C(MPP)}}
\]

Fig. 13 compares the calculated, simulated, and measured \( P_{\text{O}} \) across \( f_X \) at 118 mV of \( V_{\text{ESPR}} \) and 6.2 \( \Omega \) of \( R_{\text{ESR}} \). Testing accuracy and measurement noise produce an error. With up to ±5 mV of resolution and noise errors, measured \( V_{\text{O}} \) is up to ±4% off, which means calculated and simulated projections are off by a corresponding amount. Figs. 10 and 13 show that measured data is within the projected error window. The hard-switching assumption in the calculations also contributes error. But like Fig. 12 shows, calculations and simulations match, so this error is small. At the theory’s predicted \( f_{X(MPP)} \) of 700 kHz, the measured \( P_{\text{O}} \) is only 1% lower than the actual \( P_{\text{MPP}} \).

V. VALIDATION

A. Prototype

To validate the theory, a series switched resonant half-bridge power receiver prototype is built in 180 nm CMOS technology. The dead time logic in Fig. 14 inserts delays to prevent \( M_O \) and \( M_O \) from turning on at the same time and shorting \( V_{\text{O}} \) to the ground. The receiver prototype uses an off-chip coil PA6512-AE from Coilcraft that measures 4.52 \( \mu \text{H} \). The integrated resonant capacitor \( C_R \) is laser trimmed to 122 pF with ±1-pF accuracy. \( L_R \) and \( C_R \) resonate at 6.78 MHz.

The prototype IC in Fig. 15 occupies 644 \( \mu \text{m} \times 732 \mu \text{m} \) of area. The linear stage in Fig. 15 adjusts the distance \( d_X \) between the coils from 13 mm to 38 mm, so \( k_C \) is 0.07%–1.13%. Although separation is 38 mm, the coils are 4.5 radial lengths apart (where radial lengths refers to the radii of the coils), which is as far apart as some of the best inductively coupled systems can output power, like in [16], [18], [21], [24]. This 38-mm (power) transmission distance is suitable for implanted biosensors like glucose and blood-pressure sensors, since such sensors are typically implanted underneath the skin [1], [3]. The corresponding \( R_{\text{ESR}} \) varies 4.8–6.7 \( \Omega \). An FPGA controls \( t_{\text{OS}}, t_{\text{ON}} \) and \( f_X \) of the power receiver with 1.25 ns, 5 ns, and 53 kHz of resolution, respectively.

B. Calculation

Fig. 15. 180-nm CMOS die, measurement setup, and system top view.
The equivalent charge loss capacitance $C_{EQ}$ includes the $M_G$ gate capacitance $C_{GN}$, $M_O$’s gate capacitance $C_{GP}$, $M_O$’s source to well junction capacitance $C_{JSW}$, and $M_G$’s drain to substrate junction capacitance $C_{JDSUB}$:

$$C_{GN} = C_{OX} \cdot W_N \cdot L_N = 3.7 \, \text{pF}. \quad (23)$$

$$C_{GP} = C_{OX} \cdot W_P \cdot L_P = 1.4 \, \text{pF}. \quad (24)$$

$$C_{JSW} = \frac{C_{10} \cdot A_{JSW}}{\sqrt{1 - v_D / v_{HI}}} = 0.14 \, \text{pF}. \quad (25)$$

$$C_{JDSUB} = \frac{C_{10} \cdot A_{JDSUB}}{\sqrt{1 - v_D / v_{HI}}} = 0.37 \, \text{pF}. \quad (26)$$

$C_{EQ}$ also includes parasitic capacitance at the pins and the pads, which can be estimated from process and datasheets. The total $C_{EQ}$ is estimated as:

$$C_{EQ} = C_{PIN} + C_{PAD} + C_{JDSUB} + C_{JSW} + (2^2 + 1)(0.5C_{GN}) + (2^2 + 1)(0.5C_{GP}) = 18.2 \, \text{pF}. \quad (27)$$

Note as the voltage swing across the $C_{GD}$ ($-0.5C_{GN}$) of $M_G$ and $C_{GS}$ ($-0.5C_{GP}$) of $M_O$ doubles $v_B$, their equivalent capacitance counts four times as much. In total, $C_{EQ}$ includes 2.5 times of $C_{GN}$ and $C_{GP}$.

Table I summarizes the parameters, design variables and implied power in calculation, simulation, and measurement. Since the MOSFET models are no longer available from the manufacturer after the IC is fabricated, the parameters for simulations are chosen such that the simulated $R_{SWN}$, $R_{SWP}$, and $C_{EQ}$ closely match the calculation. Measured $R_{SWN}$ and $R_{SWP}$ are higher than estimated values, so the available power $P_{L(MAX)}$ is $1.3–1.7\%$ lower. The measured $p_{C(G)}$ is $12\%$ lower than the estimated value. However, the mismatch in $R_{ESR}$, $P_{L(MAX)}$, and $p_{C(G)}$ only causes $4\%$ error in $f_{X(MPP)}$ according to (20).

### C. Simulations

To evaluate $P_O$, the simulation monitors the average net current into $v_B$. $P_1'$ can be obtained from $L_R$’s voltage and current. The simulation estimates the ohmic loss $P_R$ on $R_{ESR}$ from the $i_L$ waveform. $P_1'$ minus $P_O$ and $P_R$ gives the charge loss $P_C$.

Fig. 16 plots the simulated and measured $P_O$ over the variable space by sweeping $t_{ON}$ and $f_X$ with 5 ns and 53 kHz of resolution, respectively. The measured $P_O$ maximizes at 269 $\mu$W when $t_{ON} = 25$ ns. The FPGA controller cannot respond within 5 ns, so 5 ns is the practical limit. Higher bandwidth is not necessary because, as Fig. 16 shows, $P_O$ is not very sensitive to $t_{ON}$ near $P_O$’s maximum power point: $P_O$ varies $2\%$ with $\pm 5$-ns variations in $t_{ON}$.

### D. Measurements

The induced $v_E$ is measured as the open-circuit voltage across $L_R$. $R_C$ models the receiver’s damping effect on the transmitter’s reflected source. So when short-circuiting $v_E$ and $R_C$, $R_C$ consumes the same power that $v_S$ supplies when the transmitter is unloaded. $v_{E(PK)}$ and $R_C$ in measurements are therefore $L_R$’s peak open-circuit voltage and the equivalent resistance that burns $v_S$’s power when the transmitter is unloaded [11]. Efficiency is $12\%–59\%$ when the coupling coefficient $k_C$ is $0.07\%–1.13\%$. This is okay because the aim is to maximize $P_O$ (not efficiency), so that $P_O$ is at its MPP.
E. Deviation

Fig. 18. Optimal duration and number of cycles between transfers.

Fig. 19. \( P_0 \) at the predicted MPP setting and the actual MPP setting

Table II summarizes and compares the calculated MPP settings and power with simulation and measurement when \( k_C > 0.15\% \). The theory’s predicted \( t_{ON(MPP)} \) is 17–30% lower than the simulation and 2–24% lower than the measurement, while the predicted \( f_X(MPP) \) is 21–66% higher than the simulation and 2–32% higher than the measurement. Testing accuracy and noise produces an error in \( v_E \) that, along with the approximations in (35) and (38), offsets projected \( f_X(MPP) \) and by translation, \( t_{ON(MPP)} \) from their actual values by up to –30% and +66%. \( P_O \), however, is still within 1.3% of its maximum power point \( P_{OMPP} \) because \( P_O \) (in Fig. 16) is fairly insensitive to settings near \( P_{OMPP} \). These inaccuracies in \( f_X(MPP) \) and \( t_{ON(MPP)} \) are therefore acceptable. In practice, rather than the accuracy of the MPP settings, it is more important that \( P_O \) at the predicted setting is as close to the actual \( P_{OMPP} \) as possible, so the MPP error is low. The MPP error is within 3.8% for simulation and within 1.3% for measurement. Since the series switched resonant half-bridge outputs as much or more power than other receivers, the \( P_{OMPP} \) theorized here is also the highest \( P_{OMPP} \) a receiver can output.

![Graph](image-url)

![Graph](image-url)

This paper explores and theorizes the MPP operation of the switched resonant half-bridge power receiver. The theory predicts the optimal phase, duration, and frequency of energy transfer in a closed form fashion. To prove the theory, a power receiver prototype is fabricated in 0.18 \( \mu \)m CMOS technology. Measurements show that at the theory’s predicted settings, the receiver outputs more than 98.7% of the actual maximum power when the coupling is 0.15%–1.13%.

### APPENDIX OPTIMAL DERIVATIONS

#### A. Peak Capacitor Voltage

Combining (14) and (15) yields:

\[
0.5C_R \left( v_{Cl(i+1)} - v_{Cl(i)} \right) \left( v_{Cl(i+1)} + v_{Cl(i)} \right) = P_{L(MAX)} \left[ \frac{v_{Cl(i)}}{2v_{C(OPT)} + v_{Cl(i)}} \right] t_{OS} \quad (28)
\]

Since the quality factor \( Q_R \) of the LC is normally much greater than one, oscillation growth from cycle to cycle is slow. Assuming \( v_{Cl(i+1)} + v_{Cl(i)} \approx 2v_{Cl(i)} \) (28) can be simplify ed as:

\[
v_{Cl(i+1)} - v_{Cl(i)} = \left( \frac{t_{OS} P_{L(MAX)}}{2C_R v_{C(OPT)}^2} \right) \left( 2v_{C(OPT)} - v_{Cl(i)} \right) \quad (29)
\]

Replacing \( P_{L(MAX)} \) and \( v_{C(OPT)} \) with expressions in (2) and (5) yields (18). Also, equation (18) can be re-written as:

\[
v_{Cl(i+1)} - 2v_{C(OPT)} = \left( 1 - \frac{\pi}{Q_R} \right) \left( v_{Cl(i)} - 2v_{C(OPT)} \right) \quad (30)
\]

With (30), \( v_{Cl(PK)} \) can be iteratively calculated as in (17).

#### B. Nonlinear Loss

Replacing \( v_{Cl} \) in (6) with (17), \( P_{NL(I)} \) can be expressed as:

\[
P_{NL(I)} = P_{L(MAX)} \left( 1 - a^{-m} \right)^2 \quad (31)
\]

Substituting \( P_{NL(I)} \) in (7) with (31) gives the closed form \( P_{NL} \):

\[
P_{NL} = \frac{1}{N_S} \sum_{i=1}^{N_S} P_{L(MAX)} \left( 1 - 2a^{-m} + a^{-2m} \right) \quad (32)
\]

At the local maximum point, intuitively, \( v_{Cl(PK)} \) centers around \( v_{C(OPT)} \). Therefore, it is fair to assume the nonlinear loss of the first cycle approximately equals that of the last cycle.
\[ P_{NL(1)} \approx P_{NL(N_S)} \]  

which gives:

\[ P_{L(MAX)} \left(1 - a^{-m} \right) \approx P_{L(MAX)} \left[1 - a^{N_S - m} \right]. \]  

Simplifying (34) yields:

\[ a^{-m} = \frac{2}{a + a^{N_S}} \approx \frac{2}{1 + a^{N_S}}. \]  

Substituting \( a^{-m} \) in (32) with (35), \( P_{NL} \) at the local maximum point can be re-written as:

\[ P_{NL(MPP)} \approx P_{L(MAX)} \left[1 - \frac{4a^{0.5N_S} - a^{0.5N_S}}{1 - a^{-2}} \left(a^{0.5N_S} + a^{0.5N_S} \right) \right]. \]  

(36)

Equation (18) can be easily derived from (36).

C. Skipped Cycles

Expanding (19) with expressions in (8) and (18) yields:

\[ P_{L(MAX)} \left(1 + \frac{2a}{1 + a} \right) \approx P_{C(f_0)} N_S^2. \]  

Again, since \( Q_a \) is normally much greater than one, \( a \) is close to one. Using Taylor expansion, the following terms in (37) can be approximated and simplified as:

\[ a^{0.5N_S} + a^{0.5N_S} \approx \frac{2}{N_S} \left(a^{0.5N_S} - a^{0.5N_S} \right) \approx 1 - a. \]  

With the approximation, (36) can be simplified as:

\[ P_{L(MAX)} \left[\frac{4a}{1 + a} \right] \approx P_{C(f_0)} N_S^2. \]  

(39)

Again, use Taylor expansion to approximate the term in (39):

\[ -\ln a = \ln \left[1 + \left(\frac{1}{a} - 1\right)\right] \approx \frac{1 - a}{a}. \]  

Solving (39) with the approximation in (40) yields the optimal number of cycles \( N_S \) between energy transfers:

\[ N_S^3 \approx \frac{4P C(f_0)}{P_{L(MAX)}} \frac{Q_R^2}{\pi}. \]  

(41)

From (41), \( f_X(MPP) \) can be easily obtained as in (20).

REFERENCES


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