Least Lossy Piezoelectric Energy-Harvesting Charger

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Abstract—Wireless microsensors can operate indefinitely when they use ambient kinetic energy in motion to replenish the battery. Of available technologies, a switched inductor can draw the most power from a piezoelectric transducer. Power consumption, however, limits how much of that power the battery receives. This paper theorizes and shows that drawing and delivering power with an inductor from the transducer into the battery directly reduces the energy the inductor carries. With less energy, inductor current and related ohmic losses are lower. This way, ohmic losses can be up to 74% lower. This is why the switched-inductor bridge is the least lossy piezoelectric charger in the state of the art.

Keywords—Energy-harvesting charger, switched inductor, direct transfer, least loss, piezoelectric, power supply.

I. PIEZOELECTRIC-POWERED CHARGERS

Wireless microsensors embedded in hospitals, factories, cars, and humans can sense, process, and share information that save money, energy, and lives [1]. But since tiny batteries drain quickly, replenishing the battery with ambient power is often necessary [2]. Ambient kinetic energy is a good source because motion is available in many applications [3]–[4].

Reported literature shows that piezoelectric transducers draw, convert, and deliver more power than their electrostatic and electromagnetic counterparts [5]–[13]. Research also shows that switched inductors (SL) draw more power than bridges with fewer power-consuming stages [5]–[8]. These SL chargers essentially collect the charge that the piezoelectric current \( i_{PZ} \) in Fig. 1 delivers and that the piezoelectric capacitance \( C_{PZ} \) collects.

Unfortunately, series resistance \( R_{ESR} \) in the switches and inductor burn some of this energy. So of the energy \( E_C \) that \( C_{PZ} \) collects across half cycles in Fig. 2 when \( C_{PZ} \)'s voltage \( v_C \) peaks, the battery receives with \( E_B \) what \( R_{ESR} \) avails:

\[
E_C = 0.5C_{PZ}v_{C(PK)}^2
\]

(1)

\[
E_B = E_C - E_{LOSS}
\]

(2)

where \( E_{LOSS} \) is the power consumed by the SL. Although CMOS switch resistance \( R_{SW} \) is usually less than 100 m\( \Omega \), \( E_{LOSS} \) is nevertheless significant because inductor resistance \( R_L \) in millimeter inductors is normally 1–10 \( \Omega \) [14].

This paper theorizes and shows with simulations how to minimize this ohmic loss. Sections II–IV describe and analyze the different ways that a SL can draw and deliver energy from a capacitive source into a battery. Section V then compares and derives which method outputs the most power and identifies which piezoelectric charger in the state of the art is, as a result, the least lossy.

II. INDIRECT TRANSFER

A. Switching Configuration

The first configuration of switched-inductor is indirect. The transfer starts with \( C_{PZ} \) energizing the inductor \( L_X \), as shown in Fig. 3. When \( C_{PZ} \) is completely discharged, \( L_X \) with \( i_{L(PK)} \) is disconnected to \( C_{PZ} \) and instead connected to \( v_B \). Notice that \( L_X \) receives all of the energy that from \( C_{PZ} \) and \( C_{PZ} \) never directly delivers its energy to the battery.

\[
E_{L} = 0.5L_Xi_{L(PK)}^2 = E_C,
\]

(3)

\[
i_{L(PK)} = \frac{C_{PZ}}{L_X}v_{C(PK)}.
\]

(4)

Therefore, the energizing time is a quarter cycle:

\[
t_e = 0.25L_Xi_{L(PK)} = 0.5\pi\sqrt{L_XC_{PZ}}.
\]

(5)

Then the inductor drains linearly to the battery with a slope of \( v_B/L_X \), and the draining time is

\[
t_d = \left( \frac{L_X}{v_B} \right)^{i_{L(PK)}}.
\]

(6)

From the current waveform and the transfer time, the ohmic loss of the total series resistance \( R_{ESR} \) is

\[
E_R = R_{ESR}i_{L(RMS)}^2 = R_{ESR}i_{L(RMS)}^2(t_e + t_d)
\]

(7)

Fig. 1. Piezoelectric-powered energy-harvesting charger.

Fig. 2. Piezoelectric voltage in a switched-inductor charger.
where \( i_{L(RMS)} \) is the root-mean-square (RMS) current across the transfer, and \( t_x \) is the total transfer time. A more detailed loss analysis can be found in Section V.

### III. INDIRECT–DIRECT TRANSFERS

#### A. Switching Configuration

An alternative way to transfer the energy from \( CPZ \) to \( v_B \) is the indirect–direct transfer. As shown in Fig. 5, \( CPZ \) first energizes \( LX \), but stops before \( CPZ \) is completely drained. Instead, it has drain initial voltage \( v_{DI}' \) when \( LX \) starts to drain, and both \( CPZ \) and \( LX \) drain directly into the battery to finish the transfer. The controller controls the energizing time so that \( CPZ \) and \( LX \) hold just the right amount of energy to drain, and both \( CPZ \) and \( LX \) drain directly into the battery to finish the transfer. The controller controls the energizing time so that \( CPZ \) and \( LX \) hold just the right amount of energy to drain at the same time at the end of the transfer. \( CPZ \) delivers the drain initial energy \( E_{C(DI)'} \) directly to the battery, while the rest of the energy, the drain initial energy on the inductor \( E_{L(DI)'} \), is delivered indirectly with the inductor. Therefore, it’s called indirect–direct transfer.

\[
\text{Energize } LX \quad \text{Drain } LX
\]

![Fig. 5. Inductor phases for indirect-direct transfers.](image)

#### B. Inductor Current

The inductor current in indirect-direct transfer, \( i_L' \), in the solid trace in Fig. 4, starts out the same as in indirect. However, since the energizing time \( t_E' \) is shorter, the inductor current only reaches \( i_L(PK)' \), which is less than \( i_L(PK) \). Assuming a sinusoidal transfer, the energy between \( CPZ \) and \( LX \) before and after \( t_E' \) is equal:

\[
E_C = E_{C(DI)'} + E_{L(DI)'} = 0.5C_vvC(DI)'^2 + 0.5L_vvL(PK)'^2 \tag{8}
\]

where \( E_{C(DI)'} \), \( E_{L(DI)'} \), \( v_{C(DI)'} \), and \( i_{L(PK)'} \) denote the drain initial energy on \( CPZ \), the initial energy on \( LX \), the voltage across \( CPZ \), and the current on \( LX \) at the beginning of the drain phase. The energizing time is the fraction of the cosine that drops the voltage across \( CPZ \) from \( v_{C(PK)} \) to \( v_{C(DI)'} \):

\[
t_E' = \frac{t_L}{2\pi} \cos^{-1}\left(\frac{v_{C(DI)'}}{v_{C(PK)}}\right) \tag{9}
\]

Next, both \( CPZ \) and \( LX \) drain into \( v_B \) across drain time \( t_D' \). Fig. 6 depicts the voltage transfer \( v_C \) with the solid line, and the steady-state extrapolation of the oscillation between \( CPZ \) and \( LX \) with the dashed trace. The capacitor voltage \( v_C \) is a sinusoidal waveform centered around the dc voltage \( v_{DC} \) with peaks of \( v_{DC} + v_{PK} \) and \( v_{DC} - v_{PK} \). In this case, the dc voltage is the battery voltage, \( v_{B} \), and since the negative peak is 0 V, the peak voltage of the sinusoid, \( v_{PK} \), is \( v_{B} \). As \( v_C \) drains from \( v_{C(DI)'} \) to 0, the drain time \( t_D' \) is the fraction of the cosine it takes to go from \( v_{B} - v_{C(DI)'} \) to \( v_{B} \):

\[
t_D' = \frac{t_L}{2\pi} \cos^{-1}\left(\frac{v_{B} - v_{C(DI)'} \right)}{v_{B}} \tag{10}
\]

Note that the direct phase of the transfer \( t_D' \) (0.33 \( t_L \) to 0.50 \( t_L \)) in Fig. 6 corresponds with the \( t_D \) in Fig. 4 (3.14 \( \mu s \) to 6.28 \( \mu s \)). Since the voltage is centered at \( v_{B} \), the energy in the LC tank would need to refer to the dc voltage. When drain phase starts, the inductor has drain initial energy \( E_{C(PK)'} \), and \( CPZ \) has referred drain initial energy \( \Delta E_{C(DI)'} \), which is \( E_{C} - E_{C(DI)'} \) from (8). As the transfer ends, the inductor is drained, so \( CPZ \) holds all the energy in the LC tank \( \Delta E_C \). For a sinusoidal transfer, the energy in the LC tank is constant:

\[
\Delta E_{C}' = 0.5C_vvC'^2 = \Delta E_{C(DI)'} + E_{L(DI)'} = \Delta E_{C(DI)'} + E_{L(DI)'} \tag{11}
\]

\[
\Delta E_{C}' = \frac{0.5C_vvC'^2}{2v_{B}} \tag{12}
\]

From (11), the exact \( v_{C(DI)'} \) to stop energize can be solved for:

\[
v_{C(DI)'} = \frac{v_{C(PK)'^2}}{2v_{B}} \tag{12}
\]

With \( v_{C(DI)'} \), the exact \( t_E' \) and \( t_D' \) to complete the transfer can be solved for with (9), (10), and (12). Similar to (7), the ohmic loss across an indirect–direct transfer is

\[
E_L = R_{ESR}i_{L(RMS)}'^2 \tag{13}
\]

where \( i_{L(RMS)}' \) is the RMS current across this transfer, and \( t_x' \) is the total transfer time for indirect-direct. Note that during energize, \( CPZ \) drains from \( v_{C(PK)} \) to \( v_{C(DI)'} \), so \( v_{C(DI)'} \) has to be lower than \( v_{C(PK)} \). Indirect–direct can only be possible when:

\[
v_{C(PK)} \leq v_{C(DI)'} \Rightarrow v_{C(PK)} \leq 2v_{B} \Rightarrow v_{C(PK)} \leq 2v_{B} \tag{14}
\]

Another way to analyze the condition for indirect–direct is to examine Fig. 6. \( v_{C(DI)'} \) has a range between ground and the peak of the steady-state extrapolation, \( 2v_{B} \):

\[
v_{C(DI)'} \geq v_{C(PK)} \leq 2v_{B} \Rightarrow v_{C(PK)} \leq 2v_{B} \tag{15}
\]

The indirect–direct scheme can draw and deliver more energy than what the inductor carries. Therefore, \( LX \) never receives the entire energy \( E_C \), and the peak inductor current is always lower. The detailed ohmic loss analysis and comparison can be found in Section V.

### IV. DIRECT–INDIRECT TRANSFERS

#### A. Switching Configuration

Since indirect-direct transfers only work under certain conditions, another direct configuration is necessary when indirect–direct is not possible. The way to accomplish this is the direct–indirect transfer, where \( CPZ \) drains into \( LX \) and \( v_B \) to start the transfer, as shown in Fig. 7. Note that \( v_B \) has to be lower than \( v_{C(PK)} \) for this transfer to start.

As \( v_C \) drains from \( v_{C(PK)} \) to \( v_B \), \( LX \) energizes. The first phase of the draining \( LX \) has the same switching
configuration as the energizing phase, so that both CPZ and LX can both drain into vB, until CPZ has no charge. At that point, LX holds draining current \( i_{L(D1)}'' \), and is connect to the battery in the second drain phase to complete the transfer. CPZ first delivers part of the energy directly to the battery in energize and drain 1 phase, then the rest of the energy is delivered with the LX in drain 2 phase. Therefore it’s called direct–indirect transfer.

\[
\text{Energize LX + Drain LX 1} \quad \text{Drain LX 2} \quad \begin{array}{c|c|c|c|c}
\hline
& \text{LX} & \text{CPZ} & \text{ESR} & \text{ESR} \\
\hline
\text{vC} & + & - & - & - \\
\text{vB} & - & + & + & + \\
\text{vDC} & + & + & + & + \\
\hline
\end{array}
\]

B. Inductor Current

The inductor current \( i_L'' \) for direct–indirect is shown in the solid trace in Fig. 8. As a reference, the indirect current \( i_L \) is also plotted with the dashed trace. The energizing phase is still a quarter cycle of the LC oscillation,

\[
t_E'' = 0.25t_{LC}.
\]

However, since the energizing voltage for LX is \( v_C - v_B \) instead of \( v_B \), the peak current \( i_{L(PK)}'' \) is lower than \( i_{L(PK)} \). The first drain phase continues the LX oscillation with \( v_C \) centered at \( v_B \), and together with the energizing phase make up the direct part of the transfer, as highlighted by the gray region in Fig. 8.

\[
t_{D1}'' = t_{D2}'' = t_{D1} + t_{D2} = \frac{v_B}{v_{C(PK)} - v_B}.
\]

The time it takes for \( v_C \) to drain from \( v_B \) to 0 is:

\[
t_D'' = \frac{t_{LC}}{2\pi} \sin^{-1} \left( \frac{v_B}{v_{C(PK)} - v_B} \right).
\]

Because the voltage is centered at \( v_B \), the energy in the LC tank would also need to refer to its dc voltage. At the beginning of the transfer, \( CPZ \) hold all the energy in the tank, \( \Delta E_{LC''} \). At the end of the direct phase, \( LX \) has drain energy \( E_{L(D1)}'' \), while \( CPZ \) has the referred energy \( \Delta E_{C(D1)}'' \). Therefore,

\[
\Delta E_{LC}'' = 0.5C_{PZ} \left( v_{C(PK)} - v_B \right)^2 = \Delta E_{C(D1)}'' + E_{L(D1)}'' = 0.5C_{PZ} v_B^2 + 0.5L_{X} i_{L(D1)}''^2.
\]

From (18), the drain inductor current is

\[
i_L(D1)'' = \sqrt{\left( \frac{L_{X}}{C_{PZ}} \right) \left( v_{C(PK)} - 2v_B \right)}.
\]

The indirect phase of the transfer is between the inductor and the battery, and the inductor current falls linearly. Therefore, the time it takes to drain the inductor is

\[
t_D'' = \frac{v_X}{v_B} i_{L(D1)}''.
\]

With the transfer time and current profile completed, we can calculate the ohmic loss during a direct–indirect transfer.

\[
E_R'' = R_{E(str)} i_{L(RMS)}'' t_X'' = R_{E(str)} i_{L(RMS)}'' \left( t_E'' + t_{D1}'' + t_{D2}'' \right),
\]

where \( i_{L(RMS)}'' \) is the RMS current across the transfer, and \( t_X'' \) is the total transfer time. Direct–indirect draws and delivers more energy than what the inductor carries. Therefore, LX never receives the entire energy \( E_C \), and the peak inductor current is always lower.

Note from Fig. 9, in order to fully drain \( CPZ \) after drain 1, ground has a range between the negative peak of the sinusoid and \( v_B \). Direct–indirect transfer therefore has to satisfy:

\[
v_{DC} - v_{PK} = v_{B} - (v_{C(PK)} - v_{B})\]

\[
= 2v_{B} - v_{C(PK)} \leq 0
\]

The same condition can be obtained from (19), since \( v_{C(PK)} = 2v_B \) is inside of a square root and therefore to be non-negative. Interestingly, the condition for direct–indirect transfer complements that for indirect–direct, meaning no matter the relationship between \( v_{C(PK)} \) and \( v_{B} \), we can always choose either direct–indirect or indirect–direct. The only exception is when \( v_{C(PK)} = 2v_B \), and that’s when both work. In fact, that’s the condition when the entire transfer is direct.

V. COMPARISON AND ASSESSMENT

Sections II–IV explain how indirect, indirect–direct, and direct–indirect schemes transfer energy. This section discusses how their ohmic losses compare. But since the distinguishing feature is how much energy the inductor carries and both indirect–direct and direct–indirect transfers output more energy than the inductor delivers, this section combines them into one: direct transfers.
A. CMOS Implementations
The piezoelectric SL charger [7]–[8] in Fig. 10 can perform all three transfers. It’s therefore used to simulate the operation and losses for all schemes. Although the switched-inductor bridge [15] in Fig. 11 can only perform direct–indirect transfers, it requires four fewer switches. So Fig. 11 shows the least lossy way of implementing direct transfers.

![Fig. 11. Switched-inductor bridge.](image)

B. Ohmic Losses
Figure 12 compares the total transfer time and peak current for indirect and direct transfers when \( V_{C(PK)} \) is 1 V and \( V_B \) is between 0.1 and 5 V. The direct–indirect region (where \( V_B \) is less than 0.5 V) is in grey and the indirect–direct region is not. The y-axis indicates the fraction of time \( t_X' \) and \( t_X'' \) and peak current \( i_{L(PK)}' \) and \( i_{L(PK)}'' \) that direct transfers require relative to indirect transfers \( t_X \) and \( i_{L(PK)} \). This means that \( i_{L(PK)}'' \) for example, nearly matches \( i_{L(PK)} \) when \( V_B = 5 \) V.

As simulation results show, direct transfers \( t_X' \) and \( t_X'' \) require 80%–95% of the time that indirect transfers require with \( t_X \). Plus, peak currents \( i_{L(PK)}' \) and \( i_{L(PK)}'' \) for direct transfers are 50%–98% of the \( i_{L(PK)} \) that for indirect transfers. With shorter transfers and lower currents, \( R_{ESR} \) consumes less ohmic power with direct transfers than with indirect.

Figure 13 shows ohmic losses for indirect and direct transfers. The y-axis here indicates the fraction of power that \( R_{ESR} \) loses with direct transfers \( E_P \) and \( E_P'' \) relative to indirect transfers \( E_P \) when \( V_B \) is 0.1–5 V. As shown, direct transfers burn 26%–86% of the power that indirect transfers consume with the same 5 \( \Omega \). Simulation validates the theory, as shown by the gray dashed traces in Fig. 13. The maximum error is 8% for indirect transfer, and 6% for direct trans.

![Fig. 12. Transfer times and peak currents for indirect and direct transfers.](image)

![Fig. 13. Ohmic loss for indirect and direct transfers.](image)

The reduction is greatest when \( V_B \) is 0.5 V because the inductor energizes and drains with direct trans. So although transfer time \( (t_X' \) and \( t_X'') \) is not the shortest, peak inductor current \( i_{L(PK)}' \) and \( i_{L(PK)}'' \) is so low that \( R_{ESR} \) still burns the least power. In other words, the loss is lower when more of the transfer is direct. This is why the switched-inductor bridge in Fig. 11 is the least lossy in the state of the art, because it transfers more power directly than the others.

VI. CONCLUSIONS
The switched-inductor bridge is the least lossy piezoelectric charger in the state-of-the-art because of low ohmic loss. Utilizing direct transfers, the inductor can draw and deliver more energy than it carries. As a result, both transfer time and peak inductor current are reduced, and the related ohmic loss is reduced by as much as 74%. With direct transfers, more power from piezoelectric transducers that draws from ambient motion can reach the battery. This battery can therefore indefinitely power the wireless microsensors to save energy, money, and lives.

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