Generating the Highest Power with a Tiny and Distant Inductively Coupled Coil

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Abstract—Although microsystems today require less power than ever before, they still cannot fit large enough batteries to sustain them for months or years at a time. Ambient energy is appealing, but only when available, which is often not the case for embedded sensors. Transmitting power wirelessly is more practical in these applications. Tiny receivers, however, capture a small fraction of the power that a distant source can deliver. So output power is low and its effects on the transmitting coil are barely noticeable. Receivers should therefore draw as much power as possible, but only as much as breakdown voltages and power losses allow. This paper shows that, although the switched bridge can draw 27% more power than resonant bridges, synchronization and ohmic losses negate that gain. In fact, breakdown voltages limit the switched resonant bridge less than others in the state of the art. Plus, the switched resonant bridge excludes the charger that the others require for maximum output power, so with less space and less added losses, power density can be greater.

Index Terms—Wireless power transfer, inductively coupled power receiver, maximum output power, damping force, battery charger, biomedical implant, and embedded microsensor.

I. WIRELESSLY-POWERED MICROSYSTEMS

Although microwatt microsystems can sense, process, and report information [1]–[3] that can save lives, energy, and money, tiny onboard batteries cannot sustain them for long [4]. Energy in light, motion, and thermal gradients can help [5], but only when available. Unfortunately, ambient energy in embedded biomedical and structural devices is often scarce, so coupling power wirelessly is the only recourse left [6]–[8].

In these cases, feeding alternating current into a transmitting coil L_T like Fig. 1 shows produces a changing magnetic field from which a nearby receiving coil L_R can draw power. The changing flux induces an electromotive-force (EMF) voltage \( V_{\text{EMF,R}} \) that can supply power. But since \( L_R \) in embedded sensors can be centimeters away from \( L_T \), \( L_R \) receives a low fraction \( k_C^2 \) of the power \( L_T \) can deliver, which is why \( V_{\text{EMF,R}} \) is normally in millivolts [9]–[10].

Since \( V_{\text{EMF,R}} \) outputs so little power, a rectifying maximum power-point (MPP) charger draws just enough power into the battery \( V_{\text{BAT}} \) to keep the receiver at its maximum power point. \( V_{\text{BAT}} \) then caches this energy until the load requires it. The purpose of the regulator is to supply and condition the power that the load demands.

This paper assesses how the state of the art in inductively coupled power receivers can draw and output the highest power possible from tiny coils that are centimeters away from their transmitting sources. For this, Section II first reviews how to draw power from small coils. Sections III, IV, and IV then review and evaluate how bridges draw and output power. Sections V and VI end with comparisons and conclusions.

II. DRAWING ELECTROMAGNETIC POWER

To understand how \( V_{\text{EMF,R}} \) in Fig. 1 generates power, first consider that impressing \( V_{\text{EMF,R}} \)’s sinusoid across \( L_R \) produces a current \( i_L(0) \) in Fig. 2 that is 90° out of phase. This results because \( V_{\text{EMF,R}} \)'s positive half cycles raise \( i_L \) and \( V_{\text{EMF,R}} \)'s negative half cycles reduce \( i_L \) about a 0-mA median. \( V_{\text{EMF,R}} \) and \( i_L(0) \) are therefore both positive and both negative half the time and opposite polarities the other half. This means, \( V_{\text{EMF,R}} \) outputs as much power as it consumes, so output power is nil.

The only way to net output power is to reduce the phase difference between \( V_{\text{EMF,R}} \) and \( i_L \). Luckily, applying an external voltage \( V_{\text{EXT}} \) at the beginning of \( V_{\text{EMF,R}} \)'s positive half cycle like Fig. 2 shows at 0 μs energizes \( L_R \) quicker to a higher peak \( i_L \) so that applying a negative voltage after that can help reduce \( i_L \) to zero at the end of the half cycle. Similarly, applying a negative \( V_{\text{EXT}} \) when the negative half cycle begins at 0.5 μs energizes \( L_R \) to a lower peak \( i_L \) so that impressing a positive voltage after that can help raise \( i_L \) to zero at the end of the half cycle. When balanced this way, \( V_{\text{EMF,R}} \) and \( i_L \) are in phase (with the same polarity), so \( V_{\text{EMF,R}} \) only sources power.
If the system is lossless, \( V_{\text{EXT}} \) recovers the power that \( V_{\text{EXT}} \) delivers with \( i_L \). In other words, \( V_{\text{EXT}} \) receives what \( V_{\text{EMF}} \) produces. And since a higher \( V_{\text{EXT}} \) raises \( i_L \) to an even higher peak, higher \( V_{\text{EXT}} \)'s can draw more power from \( V_{\text{EMF}} \). This is why \( V_{\text{EXT}} \)'s 100 mV, \( V_{\text{EXT}} \)'s 200 mV, and \( V_{\text{EXT}} \)'s 400 mV in Fig. 2 draw 12, 38, and 61 \( \mu \text{W} \), respectively. The rectifying MPP charger in Fig. 1 must therefore apply across \( L_R \) the highest voltage possible that keeps \( V_{\text{EMF}} \) and \( i_L \) in phase.

### III. Resonant Bridges

#### A. Resonant Half Bridge

Paralleling a capacitor \( C_R \) across \( L_R \) in Fig. 3 produces a voltage \( V_C \) across \( C_R \) that, when tuned to \( f_o \), crosses zero between \( V_{\text{EMF}} \)'s half cycles (similar in phase to \( V_{\text{EXT}} \) from Fig. 2). So when first energized, \( V_{\text{EMF}} \) supplies more power than it consumes, and \( L_R \) and \( C_R \) receive and exchange that energy across subsequent half cycles. But since \( V_{\text{EMF}} \) continues to source power, \( L_R \) and \( C_R \)'s energy grows until \( D_{\text{REC}} \), which is a millivolt-drop FET can implement [11], clamps \( C_R \) to \( V_{\text{REC}} \). Past that point, \( D_{\text{REC}} \) drains \( L_R \) into \( C_{\text{REC}} \). \( C_R \) then discharges into \( L_R \), and after \( V_C \) reaches zero, \( L_R \) drains into \( C_R \) to reduce \( V_C \) to a negative peak. After that, \( C_R \) drains into \( L_R \) and \( L_R \) back into \( C_R \) until, again, \( D_{\text{REC}} \) clamps and depletes \( L_R \)’s leftover energy into \( C_{\text{REC}} \). \( C_{\text{REC}} \) collects energy this way every other half cycle [7].

![Fig. 3. Schematic and simulated response of the resonant half bridge.](image)

**Drawn Power:** Considering the effects of drawn power on a distant transmitter are minimal, the receiver should draw as much power as possible. For that, since \( C_{\text{REC}} \) collects \( V_{\text{EMF}} \)'s \( i_L \) at \( V_{\text{REC}} \), \( V_{\text{REC}} \) should be as high as possible. This is the purpose of the maximum power-point charger in Fig. 3, to draw just enough power to keep \( V_{\text{REC}} \) as high as the breakdown voltage \( V_{\text{BD}} \) allows. So to keep \( D_{\text{REC}} \) from breaking, \( C_R \)'s peak-peak voltage \( V_{\text{PP}} \) should near, but not exceed \( V_{\text{BD}} \).

Since \( V_{\text{EMF}} \) in small distant coils is very low, and \( V_C \) swings to \( V_{\text{REC}} \) and \( C_R \)'s energy at \( V_{\text{REC}} \) drains into \( L_R \) and back into \( C_R \) to swing \( V_C \) to \( V_{\text{REC}} \), \( V_{\text{EMF}} \) is a negligible part of \( V_{\text{REC}} \) in \( V_C \) [10]. This means, energy in \( L_R \) and \( C_R \) is much greater than the energy \( V_{\text{EMF}} \) delivers, so \( V_C \) swings close to \( V_{\text{REC}} \) to ensure, like \( V_{\text{EXT}} \) in Fig. 2, \( V_{\text{EMF}} \) and \( i_L \) are in phase. As a result, \( L_R \) and \( C_R \) exchange energy and \( D_{\text{REC}} \) bleeds into \( C_{\text{REC}} \) once per cycle what \( V_{\text{EMF}} \) supplies across both half cycles.

\( L_R \)'s peak energy \( E_{L_R}(PK) \) at \( i_L(PK) \) therefore matches \( C_R \)'s counterpart \( E_{C_R}(PK) \) at \( V_{C(PK)} \), so \( i_L(PK) \) and \( V_{C(PK)} \) relate:

\[
E_{\text{C}(PK)} = 0.5C_R V_{\text{C}(PK)}^2 = E_{\text{L}(MAX)} = 0.5L_R i_L(PK)^2
\]

\[
i_L(PK) = \frac{V_{\text{C}(PK)} L_R}{C_R}
\]

With \( i_L \)'s in-phase \( i_L(PK) \sin(\omega_0 t) \), \( V_{\text{EMF}} \)'s \( V_{\text{EMF}(PK)} \sin(\omega_0 t) \) is able to output EMF power \( V_{\text{EMF}}, i_L \) as \( P_{\text{EMF}} \):

\[
P_{\text{EMF}} = \frac{1}{T_0} \int_0^{T_0} V_{\text{EMF}} i_L \, dt = \frac{V_{\text{EMF}(PK)}^2 V_{\text{C}(PK)}}{2L_R C_R}
\]

But since \( D_{\text{REC}} \) suffers \( V_C \)'s total swing \( 2V_{\text{C}(PK)} \), \( D_{\text{REC}} \)'s breakdown level \( V_{\text{BD}} \) limits \( V_{\text{REC}} \) to 0.5 \( V_{\text{BD}} \). Output Power: Power losses, however, reduce how much power \( V_{\text{REC}} \) collects. Of possible losses, the ohmic power \( i_L \)'s sinusoid burns in series resistances \( R_{\text{SER}} \) is usually dominant because tiny packages and skin effect elevate \( L_R \)'s portion \( R_{\text{SER}} \) to 5–10 \( \Omega \) [12]. This overall \( R_{\text{SER}} \) is:

\[
P_R = R_{\text{SER}} i_L(RMS)^2 = \frac{1}{T_0} \int_0^{T_0} V_{\text{EMF}(PK)} V_{\text{C}(PK)} \left( \frac{L_R}{C_R} \right) \left( \frac{C_R}{L_R} \right)
\]

Note \( L_R \)'s quality factor \( Q_8 \) specifies \( R_{\text{SER}} \) with 2\( \pi f_o L_R / R_{\text{SER}} \).

Interestingly, \( P_{\text{EMF}} \) and \( P_R \) both climb with \( V_{\text{C}(PK)} \). The system should therefore raise \( V_{\text{C}(PK)} \) until the rise in ohmic loss \( P_R \) cancels the gain in \( P_{\text{EMF}} \), which happens when the derivative of their difference is zero and \( V_{\text{C}(PK)} \) is \( v_{\text{C}(PK)} \):

\[
\frac{\partial}{\partial v_{\text{C}(PK)}} \left( P_{\text{EMF}} - P_R \right) \left( v_{\text{C}(PK)} \right) \left( v_{\text{C}(PK)} \right) = 0
\]

So barring other losses and limits, maximum power \( P_o \) can be

\[
P_o = \frac{P_{\text{EMF}} - P_R}{V_{\text{C}(PK)}} = \frac{V_{\text{C}(PK)}}{8R_{\text{SER}}}
\]

In practice, maximum output power is 1%–2% lower because bleeding energy from \( C_R \) clips \( V_C \)'s sinusoid to a lower peak.

#### B. Resonant Full Bridge

\( C_R \) in the resonant full bridge of Fig. 4 [6] similarly produces a voltage \( V_C \) across \( C_R \) that, when tuned to \( V_{\text{EMF}} \)'s \( f_o \), peaks between \( V_{\text{EMF}} \)'s half cycles. So \( V_{\text{EMF}} \) supplies more power than it consumes and \( L_R \) and \( C_R \) receive and exchange that rising energy across half cycles until millivolt diodes [11] clamp \( C_R \) to \( V_{\text{REC}} \). At that point, the diodes drain \( L_R \) into \( C_{\text{REC}} \), \( C_R \) then drains into \( L_R \) and \( L_R \) depletes back into \( C_R \) to reduce \( V_C \) to \( V_{\text{REC}} \). But since \( V_{\text{EMF}} \) supplies energy across half cycles, \( L_R \) has leftover energy that the diodes bleed into \( C_{\text{REC}} \). So after \( L_R \) depletes, \( C_R \) drains into \( L_R \), \( L_R \) drains back into \( C_R \) to raise \( V_C \) to \( V_{\text{REC}} \) and the cycle repeats. So in all, \( L_R \) and \( C_R \) exchange energy across half cycles and \( C_{\text{REC}} \) receives the energy \( V_{\text{EMF}} \) supplies when \( C_R \) clamps to \( V_{\text{REC}} \) and to \( V_{\text{REC}} \).

\( P_o \) and full bridges both resonate and clamp a \( V_{\text{C}(PK)} \) that dwarfs \( V_{\text{EMF}} \), \( L_R \) and \( C_R \) exchange about the same energy and produce the same current. So with the same \( R_{\text{SER}} \), they draw the same EMF power \( P_{\text{EMF}} \) and lose the same ohmic loss \( P_R \) to produce the same maximum power \( P_o \) at the same \( V_{\text{C}(PK)} \). Operationally, the only difference is that
the full bridge collects half the energy every half cycle that the half bridge collects every other half cycle.

![Diagram](image)

**Fig. 4.** Schematic and simulated response of the resonant full bridge.

Since ground diodes keep terminal voltages from dipping below 0 V, no terminal voltage in the circuit exceeds \(v_{\text{C(pk)}}\)'s \(v_{\text{REC}}\). As a result, \(v_{\text{C(pk)}}\) can be as high as breakdown level \(V_{\text{BD}}\). This means, \(v_{\text{C(pk)}}\) can be \(2\times\) that of the half bridge. By the way, if \(v_{\text{C(pk)}}\) exceeds a MOS threshold voltage, cross-coupled FETs can replace \(D_{G}^+\) and \(D_{G}^-\) or \(D_{O}^+\) and \(D_{O}^-\) [13].

IV. SWITCHED BRIDGES

A. Half-Switched Bridge

Replacing ground diodes in the resonant bridge with synchronous switches like Fig. 5 shows allows \(L_r\) to collect energy across half cycles and drain between half cycles [9]. For this, \(S_{G}^+\) and \(S_{G}^-\) close to energize \(L_r\) from \(v_{\text{EMF,R}}\) across half cycles. Then, near the end of the positive half cycle, \(S_{G}^+\) opens, and since \(L_r\)'s \(i_L\) flows up at that time, \(D_{O}^+\) drains \(L_r\) into \(v_{\text{BAT}}\). At the end of the other half cycle, \(i_L\) flows in the opposite direction, so \(S_{G}^+\) opens and \(D_{O}^-\) depletes \(L_r\) into \(v_{\text{BAT}}\).

![Diagram](image)

**Fig. 5.** Half-switched bridge.

The advantage of this configuration is that \(v_{\text{BAT}}\) has no bearing on \(i_L\) or \(v_{\text{EMF,R}}\)'s EMF power, so the system can charge \(v_{\text{BAT}}\) directly. This feature, however, is also its drawback because \(v_{\text{EMF,R}}\) is so low that \(L_r\)'s \(i_L\) is never far from zero. With such a low current, \(v_{\text{EMF,R}}\) cannot supply much power.

B. Fully Switched Bridge

The aim of the fully switched bridge in Fig. 6 is to raise \(L_r\)'s \(i_L\) to the highest level possible [10, 14]. For this, the switches connect a \(v_{\text{REC}}\) that is much greater than \(v_{\text{EMF,R}}\) to \(L_r\). Connecting \(v_{\text{REC}}\) across \(v_{\text{EMF,R}}\)'s first quarter cycle raises \(i_L\) to a high peak: to \(i_{L(pk)}\) or \(0.25v_{\text{REC,T0}}/L_r\). Reversing \(v_{\text{REC}}\) after that for another quarter cycle reduces \(i_L\) to zero and to \(-i_{L(pk)}\) across yet another quarter cycle so that reversing \(v_{\text{REC}}\) can again raise \(i_L\) to zero at the end of the cycle. This way, \(v_{\text{EMF,R}}\) and \(i_L\) are in phase and symmetrical about zero every quarter cycle. Since \(v_{\text{REC}}\) dwarfs \(v_{\text{EMF,R}}\)-connecting \(v_{\text{REC}}\) across that time raises \(i_L\) at a rate of \(v_{\text{REC}}/L_r\). With this changing \(i_L\), \(v_{\text{EMF,R}}\)'s \(v_{\text{EMF,R}}(pk)\sin(\omega T_0)\) supplies

\[
P_{\text{EMF}} = \frac{1}{0.25T_0} \int_0^{v_{\text{REC}}} \left( \frac{v_{\text{REC}}}{L_r} \right) dt = \frac{v_{\text{EMF,R}}(pk)v_{\text{REC}}T_0}{\pi L_r}.
\]

(7)

The role of the maximum power-point charger is to keep \(v_{\text{REC}}\) at the highest possible level, near breakdown voltage \(V_{\text{BD}}\).

![Diagram](image)

**Fig. 6.** Schematic and simulated response of the fully switched bridge.

Output Power: Power losses, however, reduce how much power \(C_{\text{REC}}\) collects. Of these, series ohmic power is usually dominant because tiny packages and skin effect keep \(L_r\)'s \(R_{\text{ESR}}\) near 5–10 Ω [12]. Since \(i_L\)'s triangular RMS current is \(i_{L(pk)}\sqrt{3}\) and \(v_{\text{REC}}\) is nearly \(0.25V_{\text{REC}}/L_r\), \(R_{\text{SER}}\)'s loss is

\[
P_r = \frac{R_{\text{SER}}i_{L(pk)}^2}{2} = \frac{v_{\text{REC}}^2}{L_r} \left( \frac{0.25V_{\text{REC}}/L_r}{\sqrt{3}} \right)^2.
\]

(8)

Interestingly, \(P_{\text{EMF}}\) and \(P_r\) both climb with \(v_{\text{REC}}\). The system should therefore raise \(v_{\text{REC}}\) until the rise in \(P_r\) cancels the gain in \(P_{\text{EMF}}\), which happens when the derivative of their difference is zero and \(v_{\text{REC}}\) is \(v_{\text{REC}}\): 

\[
\frac{\partial(P_{\text{EMF}} - P_r)}{\partial v_{\text{REC}}} \bigg|_{v_{\text{REC}} = v_{\text{REC}}} = 0.
\]

(9)

So barring other losses and limits, maximum output power \(P_o\) can be

\[
P_o = (P_{\text{EMF}} - P_r) \bigg|_{v_{\text{REC}} = v_{\text{REC}}} = \left( \frac{v_{\text{EMF,R}}(pk)^2}{2R_{\text{SER}}} \right)^{\frac{12}{\pi}}.
\]

(10)

[14], however, sacrifices one of every 13 cycles to synchronize the switches to \(v_{\text{EMF,R}}\)'s quarter cycles, so maximum output power can be 7.7% lower than \(P_o\).

V. SWITCHED RESONANT BRIDGE

The switched resonant bridge in Fig. 7 [12] closes \(S_r\) to, like the resonant bridge, resonate \(L_r\) and \(C_R\); but unlike the resonant bridge, sometimes opens \(S_R\) to drain \(L_r\) and \(C_R\) in series. Like before, \(C_R\) produces a voltage \(v_{C}\) that, when tuned to \(v_{\text{EMF,R}}\)'s \(i_0\), peaks between \(v_{\text{EMF,R}}\)'s half cycles. So \(v_{\text{EMF,R}}\) supplies more power than it consumes and \(L_r\) and \(C_R\) receive and exchange that rising energy across half cycles to the point \(v_{C}\) dwarfs \(v_{\text{EMF,R}}\). When \(v_{\text{C(pk)}}\) reaches its maximum power point, \(S_R\) opens and the diodes drain \(L_r\)'s leftover energy into \(v_{\text{BAT}}\). \(S_R\) then closes to drain \(C_R\) at \(v_{\text{C(pk)}}\) into \(L_r\) and \(L_r\) back into \(C_R\) so \(v_C\) reaches \(-v_{\text{C(pk)}}\), past which point \(S_R\) opens. But
since $v_{EMF}$ supplied $L_R$ across this time, the diodes deplete $L_R$'s leftover energy into $V_{BAT}$. $S_R$ then closes to drain $C_R$ into $L_R$ and $L_R$ back into $C_R$ until $V_C$ reaches $v_{C(PK)}$. After that, $S_R$ opens and $L_R$ again drains into $V_{BAT}$ to start another cycle.

![Fig. 7. Switched resonant bridge.](image)

**Power:** Since switched and non-switched resonant bridges resonate to a voltage that dwarfs $v_{EMF}$, $L_R$ and $C_R$ exchange about the same energy and produce the same current. So with the same $R_{SER}$, drawn EMF power and lost ohmic power match to produce the same maximum power at the same peak voltage. An MPP charger, however, is not necessary because the bridge's rectified output does not limit $V_C$'s swing. And since $S_R$ and the diodes keep all terminal voltages between 0 V and $V_{BAT}$, $V_C$ can swing as high as $C_R$ allows. So when $C_R$ is off chip, $V_C$ can swing to $C_R$'s off-chip breakdown level, which can be considerably greater than that of $S_R$ and the bridge [15].

**VI. COMPARISON**

A power receiver draws the most EMF power from a tiny under-damped coil when $i_L$ is in phase with $v_{EMF}$ and at the highest possible level. For this, $C_R$ in resonant bridges impresses across $L_R$ a sinusoidal voltage that peaks to $v_{REC}$. But since $C_{REC}$ in a switched bridge keeps $L_R$'s voltage steady at $v_{REC}$, $i_L$ is higher in the switched bridge, so $P_{EMF}$ in Fig. 8 is 27% higher at 2–15 V than for resonant bridges.

![Fig. 8. Simulated EMF power drawn.](image)

Since all resonant bridges resonate the same way, drawn, lost, and output power all match, so output power maxes at the same level with the same peak voltage, at 1.6 mW with 6.2 V in Fig. 9. Although the switched bridge draws more EMF power, its higher current burns more ohmic power to limit output power to about the same level.

![Fig. 9. Simulated output power.](image)

**VII. CONCLUSIONS**

Although all resonant and switched bridges can output about as much power, breakdown voltages and losses limit the switched resonant bridge to a lesser extent than others. Plus, removing the MPP charger that others require for maximum power saves space. Outputting higher power with less volume this way is crucial when space is scarce and coils are centimeters apart. True, when within millimeters, the receiver can over-damp the transmitter before losses can limit the receiver. So short a distance, however, is rare for embedded microsensors.

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