Energy Budget and High-Gain Strategies for Voltage-Constrained Electrostatic Harvesters

Erick O. Torres, Graduate Student Member, IEEE, and Gabriel A. Rincón-Mora, Senior Member, IEEE
Georgia Tech Analog, Power, & Energy IC Research Lab
Email: ertorres@ece.gatech.edu, rincon-mora@ece.gatech.edu

Abstract—Wireless micro-sensors and similar technologies must derive their energy from micro-scale sources (e.g., thin-film Li Ions, etc.) to function in volume-constrained environments like the human body. Unfortunately, confining the source to small spaces limits the total energy available to such an extent that operational life is often impractically short. Ambient energy offers an alternate and virtually boundless source, except small volumes restrain harvesting power. Voltage-constrained electrostatic CMOS harvesters, for example, draw energy from the work done against the mechanical plates of a MEMS variable capacitor at relatively slow rates, producing low output power. This paper discusses how much energy is available in such a system before and after harvesting and offers energy-conservation schemes for increasing its net energy gain (i.e., power output) during all operational phases.

I. ELECTROSTATIC ENERGY HARVESTING

Electronic micro-devices like wireless micro-sensors [1]-[2] and biomedical implants [3] must often sense and transmit information noninvasively from difficult-to-reach and volume-constrained settings where recharging on-board miniaturized fuel cells and thin-film Li Ions are impractical luxuries [4]. Harvesting ambient energy, however, offers an alternate boundless source that promises to replenish continuously what the system consumes [5]. Kinetic energy in vibrations, for instance, is abundant in many environments and applications and can be harnessed from the work done against the electrostatic force in a micro-electromechanical systems (MEMS) variable capacitor [6]-[8]. Unlike their piezoelectric [9] and electromagnetic [10] counterparts, which require exotic materials, electrostatic harvesting is CMOS compatible because the source is a MEMS capacitor [7].

Allowing vibrations to decrease the capacitance of a variable capacitor (CVAR) while keeping its voltage VC or charge QC constant produces energy in the form of charge or voltage, respectively (QC = CVAR VC). Constraining QC, however, generates unacceptably high voltages (e.g., VC ≥ 250 V) that normally exceed the breakdown limits of standard IC process technologies (e.g., ≤ 5-12 V) [11]. Clamping VC on the other hand, with the battery (VBAT) to be charged, while allowing CVAR to change, not only conveniently uses an already existing source but also limits VC to VBAT, driving harvested current directly to the battery [12].

Operationally, QC flows out of CVAR only when CVAR decreases. This characteristic implies CVAR must be clamped (i.e., pre-charged) to VBAT at its maximum capacitance point (CVAR(max)), just before CVAR decreases and drives harvesting current iHARV into the battery. As a result, drawing energy from CVAR in response to vibrations requires a pre-charge phase at CVAR(max) (Fig. 1). After the harvesting phase that ensues, when CVAR reaches its minimum point (CVAR(min)), vibrations must again increase CVAR to CVAR(max), which amounts to a reset phase.

II. ENERGY BUDGET

To separate CVAR’s plates, mechanical (vibration) force Fmechanical (Fig. 2) must exceed damping frictional and potential forces FFricition and FPotential. The electrostatic force (FE) that results in CVAR is then proportional to the initial charge (as defined by pre-charge voltage VBAT):

\[ F_E(x) = \frac{Q^2}{2\varepsilon_o A} = \frac{\varepsilon_o A V_{BAT}^2}{2x^2}, \]

where A is the total plate area, \( \varepsilon_o \) is the free space dielectric permittivity (8.85 \( \text{pF/m} \)), and x is the parallel-plate separation. As a result, the work (W) required to overcome FE or equivalently, the energy converted (ECONV) after one vibration cycle depends on FE and the resulting plate displacement (i.e., x(max) − x(min)), the latter of which translates to capacitance variation \( \Delta CVAR \):

\[ W = E_{CONV} = \int_{x_{(min)}}^{x_{(max)}} F_E(x)dx = \frac{1}{2} \Delta CVAR V_{BAT}^2. \]

The authors thank Texas Instruments for supporting and sponsoring this research.
As CVAR decreases, because vC is kept at VBAT, charge drifts out of CVAR and into the battery, producing harvesting current source iHARV of

\[
i_{HARV} = \frac{dq_{LC}}{dt} = \frac{d(C_{VAR}V_{BAT})}{dt} = V_{BAT}\left(\frac{dC_{VAR}}{dt}\right)
\]

and a harvested energy gain per cycle \(E_{HARV}\) equivalent to

\[
E_{HARV} = \int V_{BAT}i_{HARV}(t)dt = \Delta C_{VAR}V_{BAT}^2.
\]

Because CVAR remains at VBAT when it reaches CVAR(min), it retains remnant energy \(E_{REM}\) after the harvesting phase:

\[
E_{REM} = \frac{1}{2}C_{VAR_{min}}V_{BAT}^2.
\]

Recovering \(E_{REM}\) after harvesting \(E_{HARV}\), considering the battery lost \(E_{INV}\), generates a net theoretical energy gain per cycle \(E_{NET}\) in the battery of

\[
E_{NET} = -E_{INV} + E_{HARV} + E_{REM} = \frac{1}{2}\Delta C_{VAR}V_{BAT}^2.
\]

which is the energy harnessed from the environment (Eq. 2).

Considering \(E_{REM}\) is substantially low and comparable to the losses associated with transferring it, attempting to recover it offers little to no gain. It is therefore often times more efficient to lose \(E_{REM}\) by keeping CVAR open-circuited when its plates pull together. Still, a net energy gain per cycle remains after losing \(E_{REM}\):

\[
E_{NET} = -E_{INV} + E_{HARV} = \frac{1}{2}C_{VAR_{max}}V_{BAT}^2.
\]

Nevertheless, even if energy in the environment is virtually boundless, only small packets can be drawn at a time, which means power is low. The problem is that the bias, control, and power circuit pre-charging CVAR and transferring energy from CVAR to the battery require energy, possibly reducing \(E_{NET}\) to impractical levels, which is why high efficiency in all phases of operation is so important.

III. PRE-CHARGE PHASE

When considering energy-transfer strategies, pre-charging CVAR to VBAT with a switch (Fig. 3) incurs a fundamental power loss not present in inductor-based circuits because the voltage across the former does not exist in the latter. Charging CVAR(max) from zero to VBAT with a switch, for instance, irrespective of its resistance \(R\), requires a switch energy \(E_{Switch}\) that is equal to the initial investment needed in CVAR:

\[
E_{Switch} = \int V_R(t)i_R(t)dt = \frac{1}{2}C_{VAR_{max}}V_{BAT}^2 = E_{INV}.
\]

In other words, the battery loses 2\(E_{INV}\) to invest \(E_{INV}\) in CVAR (i.e., 50% efficiency).

No energy is lost (in theory), however, when channeling investment energy \(E_{INV}\) to CVAR via a transfer inductor (\(L_X\)), as shown in Fig. 4. The reason for this lossless transaction is that energizing \(L_X\) from VBAT via switch \(S_P\) does not expose \(S_P\) to a voltage because, while \(L_X\)’s current rises (\(L_X\) energizes), \(L_X\)’s switching terminal voltage \(v_{SW}\) remains within mV’s of VBAT through the entire energizing period. Similarly, de-energizing \(L_X\) into CVAR keeps the voltage across switch \(S_N\) close to zero because, while \(L_X\)’s current falls (\(L_X\) de-energizes), VSW remains within mV’s of ground. Note the pre-charge disengages during the harvesting phase so \(S_P\) and \(S_N\) shut off after the pre-charge phase terminates, analogous to a switching converter under discontinuous conduction conditions.

Key to the success of this circuit is to deliver no more and no less than \(E_{INV}\), and do so with minimal control circuits. To this end, consider that during the energizing period, both \(L_X\) and CVAR energize with a total LC energy \(E_{LC}(t)\) of

\[
E_{LC}(t) = C_{VAR_{max}}\left(V_{BAT}^2 - \cos(\omega t_{LC})\right),
\]

defined as in the Appendix, where \(\omega_{LC}\) is LC’s resonant frequency. As a result, \(E_{LC}\) reaches \(E_{INV}\) in one-sixth \(\omega_{LC}\)’s equivalent period, which corresponds to a CVAR target voltage of 0.5VBAT. In other words, \(S_P\) should engage and allow the battery to energize \(L_X\) and CVAR until CVAR charges to 0.5VBAT, after which point \(S_N\) should allow \(L_X\) to finish charging CVAR to its target of VBAT.

Note that although the energy invested by the battery \((E_{INV})\) equals the energy received in CVAR, the total charge lost by the battery does not equal the charge gained in CVAR. This difference arises because of the voltage inequality between the two, as drawing power from a larger voltage (e.g., VBAT is less than \(V_{C\text{Initial}}\)) requires less current. The total charge collected in CVAR \((\Delta q_{\text{pre}})\) as it charges from zero, for instance, is

\[
\Delta q_{\text{pre}} = C_{VAR_{max}}V_{BAT}^2\]

whereas the charge lost by the battery \((\Delta q_{\text{batاخر}})\) is

\[
\Delta q_{\text{batاخر}} = \int i_C(t)dt = \frac{1}{2}C_{VAR_{max}}V_{BAT}^2,
\]

which is half the final charge in CVAR.
vibrations across the switches are small but not zero, inducing finite conduction losses, and parasitic capacitors present require energy to charge and discharge. In the simulations results shown in Fig. 5, for example, a 3.5 V battery invests 2.75 nJ (with a peak current of 14.6 mA through 10 µH) to pre-charge 391.4 pF from 1.05 V (which had 0.22 nJ stored from the previous reset phase). What is perhaps more troubling is the control circuitry used to monitor and drive the switches because they demand power to operate. Fortunately, these power losses occur during a small fraction of the entire vibration cycle so the total energy lost is substantially low. Operating the control circuit for 125 ns (Fig. 5) at 50 µA from the 3.5 V battery would only require 22 pJ, which amounts to a negligibly small fraction of the total investment.

Nevertheless, to charge C_{VAR} fully in the presence of losses, the battery must over-invest energy. Doing so amounts to increasing the energizing target voltage (and related energizing time tE) from 0.5V_{BAT} to a higher value (e.g., 0.8V_{BAT} in Fig. 5). Circuit conditions and temperature change over time, however, shifting the ideal target in the process. A slow feedback loop that senses excess energy in C_{VAR} over several cycles and modulates the energizing target voltage corrects for the effects of changing conditions across time. For instance, comparing C_{VAR}'s voltage after the pre-charge phase (V_{CFinal}) against V_{BAT} dictates whether tE should be incrementally increased or decreased (e.g., increase tE if V_{CFinal} < V_{BAT}). In steady state, tE converges to its optimal value, ensuring C_{VAR} pre-charges to V_{BAT}, regardless of losses across the circuit and the actual value of C_{VAR(max)}.

IV. HARVESTING PHASE

When harvesting via a switch (Fig. 6(a)), vibrations separate C_{VAR}'s parallel plates (i.e., C_{VAR} decreases) and drives charge q_{HARV} (i.e., i_{HARV}) and energy E_{CHARV} into the battery:

\[ q_{HARV} = \Delta C_{VAR} V_{BAT} \]  \hspace{1cm} (13)

and

\[ E_{CHARV} = \frac{1}{2} \Delta C_{VAR} V_{BAT}^2 \]  \hspace{1cm} (14)

Note this energy is half the total energy received by the battery (E_{HARV} or \Delta C_{VAR} V_{BAT}^2, as derived earlier). The difference represents the mechanical input to the system, that is, the battery’s ideal net energy gain (E_{NET}), as previously derived.

![Fig. 6. Harvesting current via (a) an ideal switch or (b) a diode.](image)

A diode (Fig. 6(b)) reduces harvested energy gain (E_{HARV}) to

\[ E_{HARV} = q_{HARV} V_{BAT} = (V_{BAT} + v_D) \Delta C_{VAR}. \]  \hspace{1cm} (15)

which implies some of the converted energy (E_{HARV}) is diverted from the battery to overcome vD. This remains true even when considering a higher V_{C} induces a higher harvesting current (i_{HARV}) because harvesting time t_{HARV} is now shorter by the length of time C_{VAR} takes to reach C_{VAR(max)}:

\[ i_{HARV} = \frac{q_{HARV}}{t_{HARV}} = \frac{(V_{BAT} + v_D) \Delta C_{VAR}}{t_{HARV}}. \]  \hspace{1cm} (16)

As a result, vD reduces harvested energy gain (E_{HARV}) to

\[ E_{NET} = E_{HARV} - E_{INV} = E_{HARV} - \frac{1}{2} \Delta C_{VAR} V_{CFinal}^2. \]  \hspace{1cm} (17)

Pre-charging C_{VAR} to V_{BAT} + vD circumvents the brief charge-constrained event mentioned and recovers the full C_{VAR} variation, but requires a higher energy investment E_{INV} from the battery. The problem is the optimum pre-charge voltage (V_{CFinal}) of the system is V_{BAT} so V_{BAT} + vD produces a less-than-optimal net energy gain per cycle E_{NET}'. For proof, consider that differentiating E_{NET} or

\[ E_{NET} = E_{NET} - C_{VAR(min)} V_{V_BAT} \]  \hspace{1cm} (18)

with respect to V_{CFinal} equating to zero, and solving for V_{CFinal} yields V_{BAT}. A diode therefore produces a non-optimal net energy gain of

\[ E_{NET} = E_{NET} - C_{VAR(min)} V_{V_BAT}. \]  \hspace{1cm} (19)

which compared to an ideal switch case (Eq. 8), yields lower energy, as shown in Fig. 7 where a 400-100 pF C_{VAR} variation harvests 3.45 nJ and 3.69 nJ with and without the diode, respectively. The 0.7 V drop decreased C_{VAR(max)} from 400 pF to about 330 pF, reducing energy by 240 pJ per cycle (by 6.5%), which is significant considering how difficult decreasing losses at these low power levels is.

![Fig. 7. Currents and harvested energy in switch- and diode-based circuits.](image)

A synchronous transistor switch, with respect to its voltage drop, more closely resembles an ideal switch, except it requires a circuit (i.e., power) to control it and drive the parasitic capacitors it presents. The two back-to-back PMOS transistors shown in Fig. 8, for example, constitute a sample embodiment of the synchronous switch, where each device blocks the other’s body diode from conducting when disengaged [12]. Using the synchronous switch merits scrutiny and possible adoption, but only if the control circuit uses sufficiently less energy than what the diode effectively loses, which is why biasing circuits in sub-threshold is important.
V. RESET PHASE

As mentioned earlier, remnant energy \( E_{REM} \) in \( C_{VAR} \) after the harvesting phase is not large enough (pJ’s) to warrant recovering it. Simply disengaging the harvesting switch (while still disconnected from the pre-charge circuit) leaves \( C_{VAR} \) open-circuited during the reset phase, when it increases from \( C_{VAR_{(min)}} \) to \( C_{VAR_{(max)}} \). Increasing \( C_{VAR} \) under these charge-constrained conditions has the effect of decreasing \( v_C \) to a fraction of \( V_{BAT} \):

\[
V_{C_{(min)}} = \left( \frac{C_{VAR_{(min)}}}{C_{VAR_{(max)}}} \right) V_{BAT}, \tag{20}
\]

as the simulation results of Fig. 9 shows. In essence, \( C_{VAR} \)'s electrostatic force \( F_E \) now helps vibrations pull the plates together, converting some of its remnant energy \( E_{REM} \) back to the mechanical domain.

Even though this \( E_{REM} \) does not return to the battery, it can be nonetheless used and not really lost. The fact is changes in \( v_C \) indicate the state of \( C_{VAR} \), and detecting when \( C_{VAR} \) reaches \( C_{VAR_{(max)}} \) is required to start the subsequent pre-charge phase. In other words, \( E_{REM} \) is used by the control circuit, diminishing its negative impact on the system’s net energy gain per cycle. Not taking advantage of this effect increases the complexity and power requirements of the control circuit by requiring a capacitance-sensing block.

VI. CONCLUSION

Increasing the energy gain of an electrostatic voltage-constrained energy harvester amounts to understanding its energy budget and reducing the losses associated with each operational phase. Using an inductor-based pre-charger, for example, increases the pre-charger efficiency from 50% to 79%; using a transistor switch as the harvesting medium (instead of a diode) saves 240 pJ/cycle; and using the remnant energy left in \( C_{VAR} \) after the harvesting phase to sense its capacitance reduces circuit complexity and control power. Operating the control circuit in sub-threshold and regulating the pre-charge target voltage to \( V_{BAT} \) (its optimal target) over the span of several cycles (with a slow feedback loop) further optimize the system and increase its energy gain per cycle (i.e., output power). Extracting more energy from vibrations means micro-scale systems such as wireless micro-sensors and biomedical implants can replenish more of the power they consume, extending operational life in the process.