

# Maximum Power-Point Extraction of Small Switched-Inductor Piezoelectric Harvesters

Jun-Yang Lei, *Graduate Student Member, IEEE*, and Gabriel A. Rincón-Mora, *Fellow, IEEE*

Georgia Institute of Technology, Atlanta, Georgia 30332 U.S.A.

E-mail: clei8@gatech.edu and Rincon-Mora@gatech.edu.

**Abstract**—Piezoelectric harvesters are popular today because they typically draw more power from kinetic energy in motion than electrostatic and electromagnetic systems. Still, tiny transducers only derive a small fraction of what is available. Thankfully, raising the damping force with which transducers draw power increases that fraction, except overinvesting battery energy for that purpose can overdamp the system. This is why harvesters monitor output power, and current, which normally requires fast and accurate circuits that consume substantial power. This paper, however, presents a low-loss alternative. The idea is to sense how output power changes by monitoring the time that the switched inductor requires to drain its energy. This way, with readily available parameters, a piezoelectric harvester can estimate the investment that will keep the system within 2.5% of its maximum power point.

**Keywords**—Piezoelectric transducer, harvester, ambient kinetic energy, motion, maximum power point, switched inductor.

## I. HARVESTING ENERGY FROM TINY TRANSUCERS

Wireless microsensors can add performance-enhancing and energy-saving intelligence to many inaccessible and difficult-to-reach places, like hospitals, factories, farms, and others [1]–[2]. Unfortunately, the tiny batteries that these systems can afford to incorporate cannot sustain life over extended periods, which is what many applications demand. Harvesting energy from the surrounding environment, however, can.

Of available ambient sources, kinetic energy in motion is popular because vibrations in cars, airplanes, and motors are abundant and steady [3]–[4]. Piezoelectric harvesters lead the charge in this regard because they normally generate more power than their electrostatic and electromagnetic counterparts [3]. But since the damping force that miniaturized transducers impose on vibrations is miniscule, output power is very low. This is why state-of-the-art harvesters use battery energy to raise the electrical damping force in the transducer, because a higher damping force draws more power from motion [5].

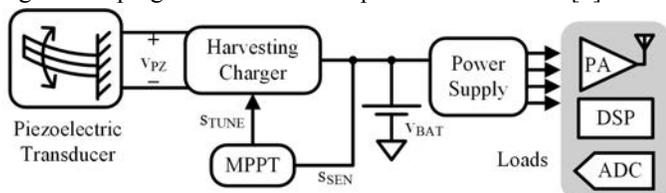


Fig. 1. Typical piezoelectric energy-harvesting microsystem.

Overdamping the system, however, is possible. If this happens, a higher damping force causes more losses than gains. Piezoelectric harvesters must therefore monitor variations in

output power, and adjust accordingly to ensure the systems remain near their maximum power point (MPP). But since vibrations are often intermittent, and tiny transducers generate little power, microsystems normally use a harvesting charger to replenish a small on-board battery. This way, a power-supply circuit like the one Fig. 1 illustrates can draw from the battery (at any time) the power that the system requires.

Monitoring how output power changes without introducing a lossy sensor into the conduction path, or without requiring precision circuits that consume considerable power, is challenging. This paper, however, shows how energy-investing switched-inductor piezoelectric harvesters can monitor variations in power from connection times that are readily available. Before introducing the scheme, though, Section II first explains the importance and operation of a switched inductor in piezoelectric chargers. Section III then describes how to monitor variations in output power from the time that this inductor requires to drain its energy. Sections IV, V, and VI finish by showing, discussing, and concluding how well this metric is able to track the maximum power point of the system.

## II. PIEZOELECTRIC CHARGERS

### A. The State of the Art

Since piezoelectric transducers generate alternating currents that produce ac voltages and batteries establish static dc voltages, many harvesters in literature use bridge rectifiers [6]–[8]. These ac–dc converters output charge when the piezoelectric voltage  $v_{PZ}$  rises above the rectified output  $v_{REC}$ . Output power is highest when the voltage drops across the switches in the network are nearly negligible [7] and  $v_{REC}$  is half the amplitude of  $v_{PZ}$ 's open-circuit voltage  $v_{PZ(OC)}$  [8].

Unfortunately, conventional bridge rectifiers suffer from two significant drawbacks. First, when operating at their maximum power point, they only collect half of the charge generated. And for this,  $v_{REC}$  should be  $0.5v_{PZ(OC)}$ . But since the battery voltage  $v_{BAT}$  is not controllable, a regulating dc–dc converter must buffer the bridge from  $v_{BAT}$ . In other words, these harvesters also require an intermediate dc–dc converter.

To overcome the first limitation, the bridge rectifier in [9] transfers the uncollected charge in the piezoelectric capacitance  $C_{PZ}$  at the end of every half cycle into an inductor that pumps it immediately back into  $C_{PZ}$ . As a result,  $C_{PZ}$  quickly discharges to ground and charges to  $v_{REC}$  in the opposite direction. Since  $v_{PZ}$  is almost always at  $v_{REC}$ , except briefly between half cycles, the system collects nearly all the charge generated.

The switched inductor in [10] overcomes the second limitation. For this, [10] lets  $C_{PZ}$  accumulate all the charge that

motion generates across every half cycle. Then, between half cycles, the system discharges  $C_{PZ}$  into an inductor that then drains the inductor into  $v_{BAT}$ . This way, without an intervening dc–dc converter, all the charge generated reaches  $v_{BAT}$ .

Still, tiny transducers only capture a miniscule fraction of the energy that is available, so output power is nevertheless low. This is why the switched inductor in [11] invests battery energy into  $C_{PZ}$ , to raise the electrostatic force with which  $C_{PZ}$  draws power from motion. Since  $C_{PZ}$  collects charge at a higher voltage,  $C_{PZ}$  draws more energy from motion. If  $v_{PZ}$  is too high, though,  $C_{PZ}$  can overdamp vibrations to the extent that output power falls. Or if the battery investment is excessive, ohmic losses can be high enough to nullify gains. To avoid these unfavorable conditions, [11] should (but does not) monitor and track its maximum power point.

### B. Energy-Investing Switched-Inductor Charger

Figure 2 illustrates the energy-investing switched-inductor piezoelectric harvester in [11]. Leakage through the transducer is normally so low that  $R_{PZ}$  hardly affects the circuit. So with zero volts to start and both switches open,  $C_{PZ}$  first collects all the charge that motion generates with  $i_{PZ}$  across its positive half cycle.  $v_{PZ}$  in Fig. 3 therefore rises and peaks to  $v_{PZ(PK+)}$  at 2 ms.

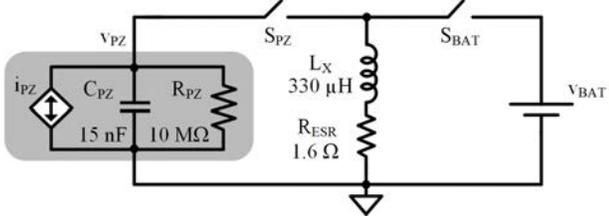


Fig. 2. Energy-investing switched-inductor piezoelectric harvester.

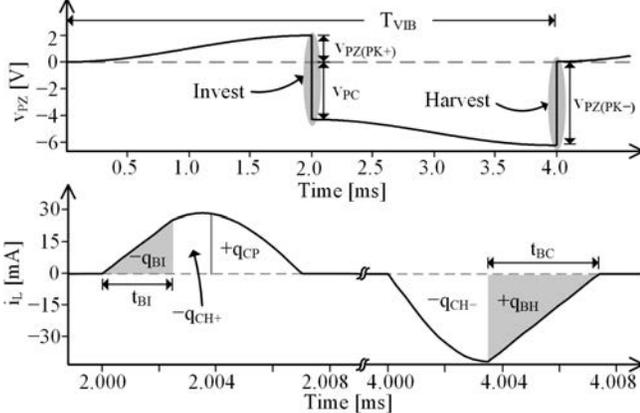


Fig. 3. Simulated time-domain waveforms.

At that point, at 2 ms, switch  $S_{BAT}$  closes to deposit battery energy into the inductor  $L_X$ .  $L_X$ 's current  $i_L$  therefore rises, and after  $t_{BI}$ , reaches 26 mA. Then,  $S_{BAT}$  opens and  $S_{PZ}$  closes to drain  $C_{PZ}$  into  $L_X$ . So as  $v_{PZ}$  falls to ground,  $i_L$  rises more to peak at 29 mA. But with  $S_{PZ}$  still closed,  $L_X$  drains into  $C_{PZ}$  in resonance fashion and  $C_{PZ}$  charges in the opposite direction to  $-v_{PC}$ .  $S_{PZ}$  opens when  $L_X$  depletes to keep  $C_{PZ}$  from draining back into  $L_X$ . In all,  $L_X$  recycles  $C_{PZ}$ 's energy and invests  $v_{BAT}$ 's energy into  $C_{PZ}$  in about 7  $\mu$ s of the 4-ms period  $T_{VIB}$ .

With both switches open,  $i_{PZ}$  deposits charge into  $C_{PZ}$  across  $i_{PZ}$ 's negative half cycle. As a result,  $C_{PZ}$  charges further in the negative direction, from  $-v_{PC}$  at 2 ms to  $-v_{PZ(PK-)}$  at 4 ms.

After this, in about 7.5  $\mu$ s,  $S_{PZ}$  first closes to drain  $C_{PZ}$  into  $L_X$  and then  $S_{PZ}$  opens and  $S_{BAT}$  closes to deplete  $L_X$  into  $v_{BAT}$ . After this, another vibration cycle begins.

### C. Maximum Power Point

The system overdamps motion when the electrostatic field across  $C_{PZ}$  couples an impeding mechanical force that is large enough to surpass the force vibrations produce in the first place [12]–[13]. But since the electromechanical coupling factor  $k_C$  of tiny devices is very low, the effects on motion are almost negligible. So when neglecting the effects of other factors, output power  $P_O$  climbs monotonically with the damping forces that increasing levels of investment energy produce.

Another possible limitation is the breakdown voltage of the chip. But with a high-voltage technology, power losses in the system subtract and limit to what extent  $P_O$  can rise. The reason for this is, switches and the series resistance of the inductor dissipate more ohmic power  $P_{LOSS}$  when they conduct more energy. In the case of the energy-investing system, extending the investment time  $t_{BI}$  that  $v_{BAT}$  requires to deposit energy into  $L_X$  raises the power that  $v_{BAT}$  delivers, the damping force that  $C_{PZ}$  establishes, and in consequence, the power  $P_{IN}$  that  $C_{PZ}$  draws from  $i_{PZ}$ . So with more power flowing through the system,  $P_{LOSS}$  in Fig. 4 rises with  $t_{BI}$ .

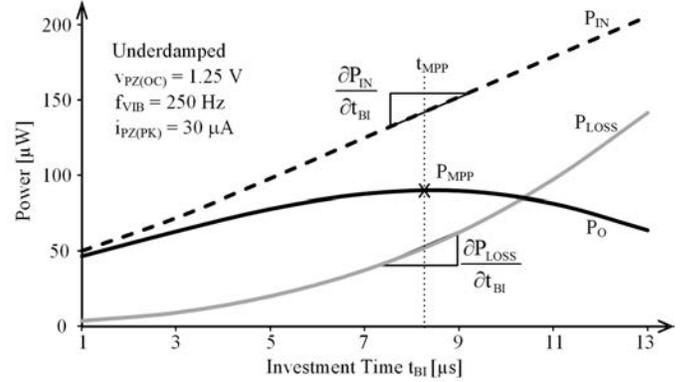


Fig. 4. Simulated input, output, and lost power across investing level.

In fact, with higher investment levels, input power  $P_{IN}$  increases linearly and ohmic losses  $P_{LOSS}$  quadratically [11]. This is unfortunate because the rise in  $P_{LOSS}$  at some point cancels that of  $P_{IN}$ . Beyond this maximum power point  $P_{MPP}$ ,  $P_O$  drops with higher investments. In other words, the system begins to overdamp after the rise in  $P_{LOSS}$  matches that of  $P_{IN}$ :

$$\left. \frac{\partial P_{IN}}{\partial t_{BI}} \right|_{P_O=P_{MPP}} = \left. \frac{\partial P_{LOSS}}{\partial t_{BI}} \right|_{P_O=P_{MPP}} \quad (1)$$

$t_{BI}$  should therefore be at the setting that balances this tradeoff, which is when the system outputs maximum power  $P_{MPP}$ .

### III. MAXIMUM POWER-POINT EXTRACTION

To track the maximum power point  $P_{MPP}$ , the system must monitor how output power  $P_O$  changes with the tuning variable  $s_{TUNE}$  in Fig. 1, or in the switched-inductor case of Fig. 2, with investment time  $t_{BI}$ . Luckily, vibrations in cars, planes, boats, and other places are steady, so  $P_{MPP}$  hardly changes across cycles. The system can therefore take several cycles to sense  $P_O$  and adjust and lock  $t_{BI}$  so  $P_O$  is near  $P_{MPP}$ .

The popular hill-climbing algorithm [14], in fact, relies on this slow time constant. With this scheme, the system raises  $s_{TUNE}$  in one cycle and monitors how  $P_O$  responds in the next. If  $P_O$  rises, the system again raises  $s_{TUNE}$ , and continues this way after each consecutive cycle until  $P_O$  finally drops. When  $P_O$  falls, which happens only after  $t_{BI}$  in Fig. 4 increments past its optimal setting  $t_{MPP}$ , the system locks or alternates between the last two settings. Since the small deviation in  $t_{BI}$  that results from switching between consecutive settings near  $t_{MPP}$  causes minimal variations in  $P_O$ ,  $P_O$  remains near  $P_{MPP}$ .

#### A. Sense Output Power

Output power  $P_O$  is the product of battery voltage  $v_{BAT}$  and the current that  $v_{BAT}$  receives as  $i_{BAT}$ . Unfortunately, sensing  $i_{BAT}$  by monitoring the voltage  $v_S$  that a series resistor  $R_S$  drops requires substantial power. If  $R_S$  is one of the switches in the network, for example,  $v_S$  is low and changes quickly. This means, the circuit used to monitor  $v_S$  must be accurate and fast, and as a result, also lossy [15]. Even if  $R_S$  is not already in the circuit, adding  $R_S$  raises ohmic losses  $P_{LOSS}$  in the circuit, so  $R_S$  and  $v_S$  should also be low. A low-pass filter across the inductor also produces a voltage that is proportional to  $i_{BAT}$ . Except, the voltage is low, sensing circuits are complex, and the filter is bulky (and normally off chip) [15].

#### B. Extract Output Power from Connection Times

Monitoring how much energy inductor  $L_X$  transfers is another way of sensing power.  $L_X$ 's current  $i_L$  is key in this respect because  $i_L$  determines how much energy  $L_X$  holds as  $E_L$  or  $0.5L_X i_L^2$ . Luckily,  $L_X$ 's voltage  $v_L$  is nearly constant through every transaction, so time  $t_X$  sets  $i_L$  to  $t_X v_L / L_X$  and  $E_L$  to

$$E_L = 0.5L_X i_L^2 = 0.5L_X \left[ \left( \frac{v_L}{L_X} \right) t_X \right]^2 = \left( \frac{v_L^2}{2L_X} \right) t_X^2. \quad (2)$$

In other words, connection time can be an indicator for power.

In the case of the energy-investing switched-inductor piezoelectric harvester in Fig. 2,  $v_{BAT}$  collects energy  $E_{BC}$  when  $L_X$  connects to  $v_{BAT}$  across collection time  $t_{BC}$  at the end of the negative half cycle (from Fig. 3). Similarly,  $v_{BAT}$  invests energy  $E_{BI}$  when  $L_X$  connects to  $v_{BAT}$  across investment time  $t_{BI}$  at the end of the positive half cycle. Output energy per cycle  $E_O$  is therefore their difference:

$$E_O = E_{BC} - E_{BI} = \left( \frac{v_{BAT}^2}{2L_X} \right) (t_{BC}^2 - t_{BI}^2) \propto t_{BC}^2 - t_{BI}^2, \quad (3)$$

where  $v_L$  is  $v_{BAT}$ ,  $P_O$  is  $E_O$  over the vibration period  $T_{VIB}$ , and  $P_O$  and  $E_O$  are both proportional to  $t_{BC}^2 - t_{BI}^2$ .

### IV. PERFORMANCE

#### A. Error

In practice,  $v_L$  is not exactly  $v_{BAT}$  across collection time  $t_{BC}$  or investment time  $t_{BI}$ . The reason for this is that parasitic resistances  $R_{ESR}$  in the conduction path drop part of  $v_{BAT}$ . Or stated differently,  $R_{ESR}$  consumes some of the energy that  $v_{BAT}$  would have otherwise invested or collected.

Luckily,  $R_{ESR}$  is so low that it drops a small fraction of  $v_{BAT}$ . 1.6  $\Omega$ , for example, causes a 0.9-mA or 1.2% error in the

73.4 mA that  $i_L$  builds over the 7- $\mu$ s span that  $v_{BAT}$  requires to invest  $E_{BI}$  in Fig. 5. Since the estimated current  $i_L'$  neglects the energy lost in  $R_{ESR}$ ,  $i_L$  is lower than  $i_L'$ . In other words, this method overestimates the investment  $E_{BI}$ .

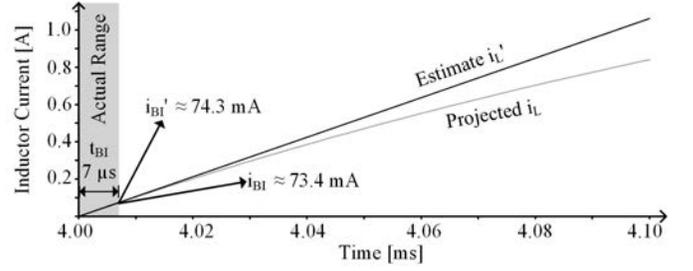


Fig. 5. Simulated and predicted investment current through the inductor.

Similarly, 1.6  $\Omega$  in Fig. 6 causes a 2-mA or 2.2% error in the 88.7 mA that  $L_X$  holds when  $L_X$  starts draining  $E_{BC}$  into  $v_{BAT}$ , which requires 8.18  $\mu$ s to exhaust. Since the estimate  $i_L'$  neglects the loss in  $R_{ESR}$ ,  $L_X$  delivers more energy than predicted by  $t_{BC}$ .  $i_L$  is therefore higher than  $i_L'$ , and  $v_{BAT}$  receives more charge than expected.

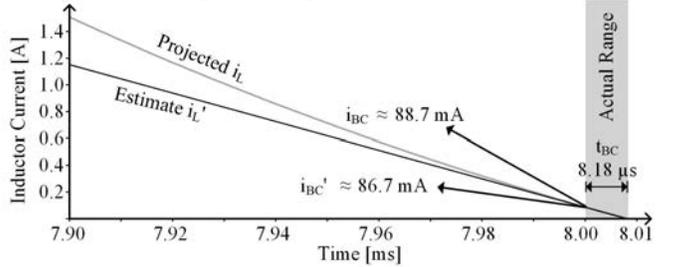


Fig. 6. Simulated and predicted collection current through the inductor.

#### B. Output Power

Because overestimating the investment and underestimating the collection are both pessimistic,  $P_O$  in Fig. 7 is higher than predicted. In fact, since  $R_{ESR}$  consumes more power when  $L_X$  transfers more energy, errors in  $i_L$  climb with higher power levels. Figures 5 and 6 show this because, with more energy,  $L_X$  requires more time to energize and drain. And with more time, the error between  $i_L'$  and  $i_L$  grows. This is why the difference between  $P_O$  and  $P_{EST}$  swells with rising investments.

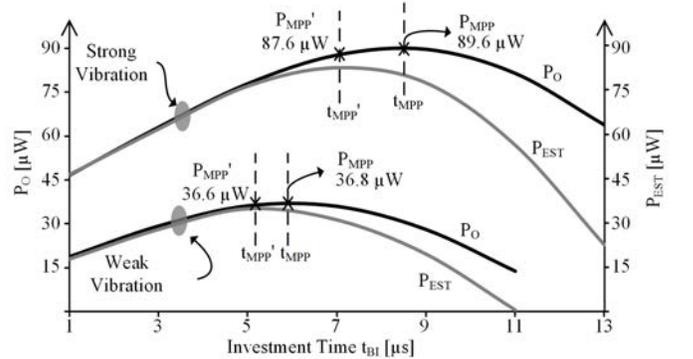


Fig. 7. Simulated and predicted output power across investment time.

#### C. Maximum Power Point

Since the error worsens with rising power levels, the prediction is more accurate with lower investments. In fact, the error near the maximum power point  $P_{MPP}$  is not significant, as Fig. 7 demonstrates. As a result, the predicted maximum power point

$P_{MPP'}$  is only 2  $\mu\text{W}$  below the 89.6  $\mu\text{W}$  peak that 30  $\mu\text{A}$  of piezoelectric current  $i_{PZ}$  across 15 nF can produce and 0.2  $\mu\text{W}$  below the 36.8  $\mu\text{W}$  that 17.5  $\mu\text{A}$  across 15 nF can generate.

Interestingly, the optimal investment time predicted  $t_{MPP'}$  is consistently lower than the actual counterpart  $t_{MPP}$ . This happens because the prediction is invariably pessimistic, and the error level fails to climb with higher investment levels. In other words, the prediction overestimates how fast losses grow. With vibrations that produce 30  $\mu\text{A}$ , for example, the predicted investment time  $t_{MPP'}$  is roughly 7  $\mu\text{s}$ , whereas the actual  $t_{MPP}$  is 9  $\mu\text{s}$ . Similarly,  $t_{MPP'}$  for 17.5  $\mu\text{A}$  is 5  $\mu\text{s}$  and  $t_{MPP}$  is 6  $\mu\text{s}$ .

As Fig. 7 shows, however, output power  $P_O$  hardly changes near its maximum power point  $P_{MPP}$ . This means, a slight deviation from the optimal investment time produces a small error in the maximum power point. This is why the maximum power-point error  $\Delta P_{MPP(E)}$  in Fig. 8 is below 2.5% for vibrations that produce up to 89.7  $\mu\text{W}$  from up to 30  $\mu\text{A}$  of piezoelectric current across 15 nF. Since the prediction is increasingly pessimistic with higher power levels, the error is less than 1% for up to 15  $\mu\text{A}$  and 1.5%–2.5% for 16–30  $\mu\text{A}$ .

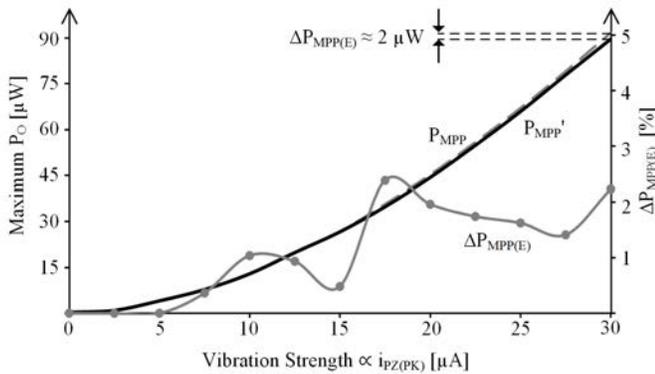


Fig. 8. Maximum output power and power point error across vibration strength.

## V. IMPLEMENTATION

Ultimately, knowing  $P_O$  or  $E_O$  is not important when hill-climbing towards  $P_{MPP}$ . What matters is how  $P_O$  or  $E_O$  changes across cycles. And since both  $P_O$  and  $E_O$  are proportional to  $t_{BC}^2 - t_{BI}^2$ , variations in this difference is sufficient to find the optimal investment time  $t_{MPP}$  with which the system produces the most output power  $P_{MPP}$ . In other words, removing  $v_{BAT}^2/2L_X$  and  $T_{VIB}$  from  $E_O$ 's and  $P_O$ 's expressions does not change the behavior or location of the peak in Fig. 7. This means, knowing the values of  $v_{BAT}$ ,  $L_X$ , and  $T_{VIB}$  is entirely unnecessary when finding  $t_{MPP}$ .

Since the tuning variable  $s_{TUNE}$  in Fig. 1 with which the harvester in Fig. 2 adjusts is the investment time  $t_{BI}$  in Fig. 3,  $t_{BI}$  is a known quantity. The only "unknown" in  $t_{BC}^2 - t_{BI}^2$  is the collection time  $t_{BC}$ . But since the controller that switches  $L_X$  in Fig. 2 sets  $t_{BC}$ ,  $t_{BC}$  is also known. So with  $t_{BC}$  and  $t_{BI}$ , the controller can decipher  $t_{MPP}$ . And since  $t_{MPP'}$  always lags  $t_{MPP}$ , the system can also offset the prediction to reduce the error.

## VI. CONCLUSIONS

The algorithm presented here can find the maximum power point of small energy-investing switched-inductor piezoelectric harvesters without sensing current, and with less than 2.5% error. The system therefore saves the power that a calibration

period or a current sensor would have otherwise required. This savings is significant because skipped cycles and circuits that sense current lose considerable power. True, typical bridge rectifiers do not use an inductor. However, switching an inductor not only avoids the overhead of a charging dc–dc buffer but also allows the system to raise the damping force in the transducer. This is important because small transducers suffer from low electromechanical coupling factors. So without energy with which to raise the damping force in the transducer, output power is very low. In other words, switching an inductor as proposed outputs the highest possible power.

## ACKNOWLEDGEMENT

The authors thank Kowshik Murali for his contributions.

## REFERENCES

- [1] R.J.M. Vullers, R.V. Schaijk, H.J. Visser, J. Penders, and C.V. Hoof, "Energy harvesting for autonomous wireless sensor networks," *IEEE Solid-State Circuits Magazine*, vol. 2, no. 2, pp. 29–38, Spring 2010.
- [2] D.A. La Van, T. McGuire, and R. Langer, "Small-scale systems for in vivo drug delivery," *Nature Biotechnology*, vol. 21, no. 10, pp. 1184–1191, Oct. 2003.
- [3] S. Roundy, P.K. Wright, and J. Rabaey, "A study of low level vibrations as a power source for wireless sensor nodes," *Computer Communications*, vol. 26, no. 11, pp. 1131–1144, 1 July 2003.
- [4] P.D. Mitcheson, E.M. Yeatman, G.K. Rao, A.S. Holmes, and T.C. Green, "Energy harvesting from human and machine motion for wireless electronic devices," *Proceedings of the IEEE*, vol. 96, no. 9, pp. 1457–1486, Sept. 2008.
- [5] R.D. Prabha, D. Kwon, O. Lazaro, K.D. Peterson, and G.A. Rincón-Mora, "Increasing electrical damping in energy-harnessing transducers," *IEEE Transactions on Circuits and Systems II*, vol. 58, no. 12, pp. 787–791, Dec. 2011.
- [6] G.A. Lesieutre, G.K. Ottman, and H.F. Hofmann, "Damping as a result of piezoelectric energy harvesting," *Journal of Sound and Vibration*, v. 269, n. 3–5, pp. 991–1001, Jan. 2004.
- [7] Y. Lam, W. Ki, and C. Tsui, "Integrated low-loss CMOS active rectifier for wirelessly powered devices," *IEEE Transactions on Circuits and Systems II*, vol. 53, no. 12, pp.1378–1382, Dec. 2006.
- [8] G.K. Ottman, H.F. Hofmann, A.C. Bhatt, and G.A. Lesieutre, "Adaptive piezoelectric energy harvesting circuit for wireless remote power supply," *IEEE Transactions on Power Electronics*, vol. 17, no. 5, pp. 669–676, Sept. 2002.
- [9] Y.K. Ramadass and A.P. Chandrakasan, "An efficient piezoelectric energy harvesting interface circuit using a bias-flip rectifier and shared inductor," *IEEE Journal of Solid-State Circuits*, vol. 45, no. 1, pp. 189–204, Jan. 2010.
- [10] D. Kwon and G.A. Rincón-Mora, "A single-inductor ac–dc piezoelectric energy-harvester/battery-charger IC converting  $\pm(0.35$  to 1.2V) to (2.7 to 4.5V)," *IEEE Int. Solid-State Circuits Conf.*, pp. 494–495, Feb. 2010.
- [11] D. Kwon and G.A. Rincón-Mora, "A single-inductor 0.35 $\mu\text{m}$  CMOS energy-investing piezoelectric harvester," *IEEE Int. Solid-State Circuits Conf.*, pp. 78–79, Feb. 2013.
- [12] P.D. Mitcheson, T.C. Green, E.M. Yeatman, and A.S. Holmes, "Architectures for vibration-driven micropower generators," *J. of Microelectromechanical Systems*, vol. 13, no. 3, pp. 429–440, June 2004.
- [13] D. Galayko and P. Basset, "A general analytical tool for the design of vibration energy harvesters (VEHs) based on the mechanical impedance concept," *IEEE Transactions on Circuits and Systems I*, vol. 58, no. 2, pp. 299–3311, Feb. 2011.
- [14] T. Eswam and P.L. Chapman, "Comparison of photovoltaic array maximum power point tracking techniques," *IEEE Transactions on Energy Conversion*, vol. 22, no. 2, pp. 439–449, June 2007.
- [15] H.P. Forghani-zadeh and G.A. Rincón-Mora, "Current-sensing techniques for dc–dc converters," *IEEE Midwest Symposium on Circuits and Systems*, vol. 2, pp. 577–580, Aug. 2002.