

Tales of the Continuum: A Subsampled History of Analog Circuits

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Prologue

Contending with the Aegean's capricious weather has been a fact of life for Greek mariners throughout recorded history. And so it was just another unremarkable October squall that delayed Dimitrios Kondos and his crew from returning home in the fall of 1900. With not much else to do while waiting for the weather to clear, the ship's team of sponge divers went to work where nature had boxed them in, off the coast of Antikythera (an isle not far from Crete). While diving about 60 meters below the surface, Elias Stadiatos was stunned by the surreal sight of life-size (and lifelike) statues seeming to reach for him through centuries of silt. His excited, near-incoherent ramblings about "a heap of dead, naked women" worried his crewmates that he'd suffered a serious problem with his air supply [1].

Another, more violent, Aegean storm had claimed a cargo vessel there two millennia earlier. Excavations yielded a bounty of treasure from the wreckage of what likely had been a ship bound for Rome. Aside from the spectacular bronze and marble statuary that had startled Stadiatos were piles of coins (whose features helped date the wreck to between 85 and 60 B.C.E.), jewelry, and the usual assortment of utensils, amphorae and other everyday objects. Almost overlooked among the debris was what appeared to be a wheel

embedded in some rock. When researchers got around to examining the artifact more closely, they found that the "rock" contained parts of a remarkably sophisticated device, now called the Antikythera mechanism. Thanks to modern imaging technology, archeologists have been able to read about 95% of the text inscribed on the components. Better yet, this same technology has enabled a reconstruction of most of the device. This work has shown the mechanism to be an orrery – a machine for illustrating the motions of the planets. So advanced is the craftsmanship that it predates by fourteen centuries any machinery of comparable complexity and precision.

The Antikythera mechanism is in fact an analog computer, where *analog* is used in the original sense of the term. The universality of physical laws often allows a problem in one domain to be reformulated as an analogous problem in another domain, where solutions might be found more readily. This universality underpins the operation of the Antikythera computer, whose hand-cranked array of some three dozen gears models celestial mechanics with physical mechanics. Aside from being conveniently smaller than a solar system, an orrery can also run its simulation forward or backward in time, allowing the prediction of important celestial events, as well as enabling a study of the past. Although we don't know whether the Antikythera mechanism was actually used for such astronomical purposes or was simply an expensive toy for a wealthy Roman patron, its mere existence is evidence of an ancient and conscious appreciation of the analog idea.

Because physical variables are continuous quantities, the use of analog computers to model real-world phenomena led to *analog*

gradually acquiring its modern association with continuity in time or amplitude. As digital computation displaced analog computation, the earlier meaning of the word faded somewhat into obscurity, and now serves mainly as the answer to a trivia question.

Introduction

Computation is one of the traditions that gave rise to modern analog electronics. Others include communication and instrumentation, and this list is by no means exhaustive. The vastness of these topics individually, to say nothing of them collectively, makes a comprehensive examination impossible. We offer this article instead in the spirit that a sub-Nyquist sampling is better than none, and present an admittedly incomplete, biased selection of some analog circuits that may fairly be deemed "classic" by virtue of their historical priority or influence on later developments. We apologize in advance for the inevitably gross errors of omission. We can aspire here only to avoid serious errors of commission.

Analog Electronics in Computation and Control

After a long gestation, the idea of analog computation re-emerged in earnest in the late 19th and early 20th centuries. An important and oft-cited example is Kelvin's *harmonic synthesizer* of 1878 [2]. The synthesizer, designed by William Thomson before becoming Lord Kelvin, was a special-purpose mechanical device (in this case, for predicting tide heights), just as was the Antikythera mechanism. Several decades later Vannevar Bush and Harold L. Hazen of MIT elaborated on many of Kelvin's ideas to develop the Differential Analyz-



a) Main fragment of Antikythera mechanism; b) A modern reconstruction. (Wikipedia, "Antikythera mechanism," retrieved 18 Sept. 2007.)

er in the early 1930s [3]. The Analyzer was the first general-purpose analog computer, and in its first incarnation was capable of solving sixth-order differential equations.

Even though the mechanical Differential Analyzer could solve complex problems considerably faster than humans, the second World War brought an urgent need for still faster computation. Appreciating that electronic means would be far more agile than the Analyzer's sluggish mechanicals, David Parkinson and Clarence Lovell of Bell Laboratories proposed in 1940 what would eventually become the M-9 Electrical Gun Director [4][5]. The M-9 development team included future EE "household names" Hendrik Bode, Claude Shannon and Richard Blackman. Operating in real time on aircraft tracking data supplied by an SCR-584 radar unit, the M-9 analog computer not only predicted trajectories, it also controlled the aiming and firing of artillery to maximize the probability of a hit. When used in conjunction with proximity-fuze equipped ordnance, these technologies reduced by over an order of magnitude the amount of ammunition required. The astonishing speed with which the art matured on several fronts simultaneously is evident from the fact that just four of 104 V-1 flying bombs launched toward London in late August of 1944 made it to their target [6]. Only two months earlier, before this equipment and trained crews could be deployed, over 80% of the fast, low-flying bombs had managed to slip through British air defenses to devastating effect.

At the heart of the gun director's computers were vacuum-tube feedback amplifiers configured to perform mathematical functions such as integration, inversion and summation. A classic paper by Ragazzini, Randall and Russell (submitted in April of 1946) describes the details of how this magic works, and in the process introduces a now-familiar term [7]:

As an amplifier so connected can perform the mathematical operations of arithmetic and calculus on the voltages applied to its

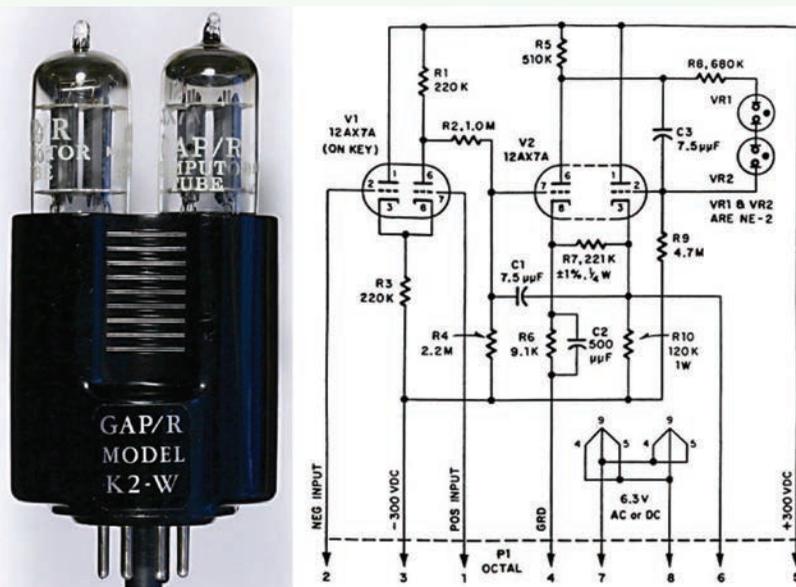


Figure 1: K2-W operational amplifier: a) Photo (courtesy of Joe Sousa, <http://www.philbrickarchive.org/>); b) Schematic (from a K2-W datasheet; courtesy of Bob Pease, National Semiconductor)

input, it is hereafter termed an "operational amplifier."

The paper also perfunctorily acknowledges L. Julie and G. A. Philbrick without detailing the nature of their contributions. Under subcontract to Philbrick (who in turn had a contract to develop an analog computer for bombing simulators), Loebe Julie had greatly simplified and otherwise improved the prototype op-amps used early in the war, and evidently passed this knowledge on to Ragazzini's co-authors [8]. In 1952 George A. Philbrick Researches went on to introduce the first commercial op-amp, the K2-W (see Figure 1), whose influence on subsequent op-amp development is incalculable.

This DC-coupled amplifier operates off of +/-300V power supplies, and manages +/-50V swings into 50k loads. Possessing a unity-gain frequency of 300kHz, a minimum DC gain of 10,000 (and typically about twice that), and a list price of \$22, the K2-W was an instant classic.

The K2-W's minimalist design reflects one of the lessons learned during the M-9's development: Keep the dynamics simple to facilitate stability. With just two stages of amplification, the K2-W satisfies that dictum. To make up for the associated tradeoff in DC gain, the

K2-W employs positive feedback around the second gain stage (through R7), boosting the gain by approximately a factor of five. The use of positive feedback here may surprise the many engineers who acquire the belief somewhere in their EE education that positive feedback is only good for making oscillators or latches and is otherwise to be avoided. The successful use of positive feedback by the K2-W powerfully refutes that unfortunately widespread misapprehension. Indeed, as we'll discuss later, not only did positive feedback precede the use of negative feedback in electronics, it in fact enabled the age of electronics to begin in earnest.

The K2-W also exploits the Miller effect to assure simple dynamics. The second stage's high voltage gain assures that the effective capacitance seen by the first stage is many times the value of C1 (imagine trying to lift an object when the other end is pulled by an opposing force; the apparent increase in weight is the essence of the Miller effect). Although this Miller multiplication of capacitance certainly reduces bandwidth, it also assures near-single pole behavior over a broad frequency range. This latter attribute is valuable for a general-purpose building block, as it allows engineers to

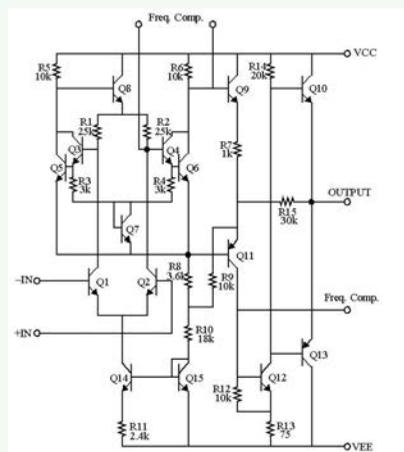


Figure 3: μ A709

had recently emerged as a standard for many discrete solid-state op-amps (e.g., the GAP/R P65).

The 709 clearly shares a great deal with its progenitor, the 702, while going well beyond it. The resistively-loaded differential input stage (Q1/Q2, biased by current source Q14 which, in turn, is slaved to Q15), performs a differential-to-single ended conversion with a slightly more sophisticated implementation of the same idea used in the 702 (here, Darlington-connected Q3/Q5 and emitter follower Q8 together comprise the “op-amp within the op-amp”). Transistors Q3 and Q4 are biased to a low current without the use of large-value area-consumptive resistors by making the voltage across current-setting resistor R3 depend on the small difference between two diode voltages (those of Q5 and Q7). This same trick synthesizes a low current in Q14 without requiring absurdly large resistor values. This clever circuit (known, sensibly enough, as a Widlar current source) is an early expression of a Widlar IC-design rule: “Replace passives by transistors wherever and whenever possible”. This philosophy remains an important guiding principle of analog IC design.

The second gain stage is a resistively-loaded common-emitter amplifier using Darlington pair Q4/Q6. Emitter follower Q9 participates in a downward level shift, in conjunction with common-base lateral PNP transistor Q11. The 709 is the first commercial product in which a lateral PNP transistor

makes an appearance (an IC for the Minuteman II missile had used one a bit earlier). Widlar’s design accommodates the dreadful characteristics of these early devices (made out of re-purposed NPN parts), which include a β that is nominally two. Widlar’s design allegedly continues to function (if one parses the word function generously) even if β is as small as 0.2.

To achieve the high open-loop gains demanded by users of op-amps, this design has a third gain stage, with Q12 in a resistively-loaded common-emitter amplifier. The presence of a third stage complicates using the 709, however, owing to the challenges of stabilizing a feedback amplifier that contains three cascaded stages. Widlar consequently makes externally accessible every high-impedance node in the op-amp to allow the user great flexibility in connecting a host of RC networks (many of them suggested by Fairchild in the 709’s data sheet and applications notes) to achieve satisfactory stability, bandwidth and settling time. Sometimes, the user even succeeded.

The output of the third gain stage drives a textbook complementary emitter follower. The PNP transistor Q13 can be (and is) implemented as a vertical PNP device, whose characteristics are better matched to those of an NPN than is a lateral PNP. A simple complementary buffer unfortunately possesses a well-known “dead zone” in its input-output transfer characteristic; there is roughly a 1.4V range of input voltages over which neither transistor conducts. Widlar employs local negative feedback around the output stage (through R15) in an effort to reduce the resulting distortion.

Fairchild’s applications notes make a game (and unintentionally amusing) attempt at moderating a user’s fears about the output driver’s robustness:

Although it is not clear from the schematic, the output stage is actually short-circuit-proof for a short period of time [10].

Murphy guarantees that *your* short circuits will always persist just a little longer than that unspec-

ified “short period of time.”

The spectacular success of the 709 quickly drove prices down as it drove production volumes up (despite yields that were simply terrible for a long time; Dave Fulgar assumed the task of solving the yield problem). This op-amp, introduced in November of 1965 at approximately \$70 (\$50 in large quantities), was the first to break through the \$10 barrier (and then the \$5 barrier by 1967), guaranteeing extremely widespread use. By 1969, op-amps were selling for around \$2. Unable to compete against exponential price reductions, the K2-W was retired in 1971, its twentieth year of continuous production.

Widlar didn’t just work on op-amps at Fairchild, he also designed a popular pair of comparators (the 710 and the 711), whose 40ns response time represents an order-of-magnitude improvement over the speeds achieved by contemporary general-purpose op-amps reluctantly impressed into service as comparators.

Widlar’s last design for Fairchild, the μ A726, rolled out in 1965. The high-precision differential pair’s on-chip temperature-controlled heater enables offset drifts of $0.2\mu\text{V}/^\circ\text{C}$ over the entire military temperature range. In two years, Widlar had put five ICs into production and firmly established analog IC design as a legitimate (and profitable) discipline.

He was just warming up.

The LM101 (1967) and LM101A (1968)

The success of the 709 emboldened Widlar to request a substantial upgrade in his compensation. When Fairchild declined to provide it, he and Talbert left the company in December of 1965 for what eventually became part of National Semiconductor. His first IC for National was a voltage regulator (the LM100). His next design was an op-amp intended to repair several shortcomings of the 709. He sought to outdo his earlier creation by providing a larger input common-mode range, lower input current, higher open-loop gain and

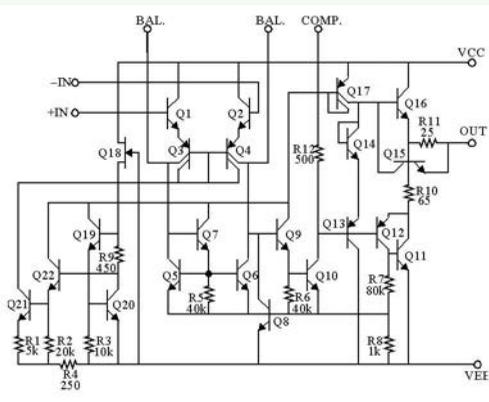


Figure 4: LM101A

simpler compensation. Finally, he wanted to protect the part against output short-circuits of arbitrary duration. The LM101 was the result, with an improved version, the LM101A, following within a year (Figure 4).

A good way to solve the level shift problem is to alternate NPN- and PNP-based stages, whose common mode shifts can cancel. Unfortunately, the poor performance of lateral PNPs normally precludes their use. Widlar's circuits manage to use these inferior PNPs in ways that mitigate their deficits to a surprising degree.

The LM101A's input stage magically mimics a PNP-based differential amplifier by combining good NPNs with the level-shifting polarity of the not-so-good PNPs. The pair of NPN input emitter followers (Q1/Q2) enable low base current, and the differential common-base configuration of PNPs (Q3/Q4) solves the level-shift problem. The fast common-base configuration also minimizes the bandwidth impact of the slow PNPs.

The LM101 is the first op-amp to use active loads, enabling much higher gains per stage, and also the first to use a mirror load to perform differential-to-single ended conversion. These remain standard analog circuit idioms, forty years after the LM101's debut.

The second stage similarly achieves high gain, thanks to current-source load Q17. Two level-shifting devices, Q13 and Q14, function to bias the complementary emitter follower (Q11/Q12/Q16) to avoid the dead zone problems of the 709.

Again because of the poor char-

acteristics of PNPs, the output follower departs somewhat from standard textbook configurations: the pulldown device is a *compound PNP* (also known as a complementary Darlington pair) – a combination of an NPN (Q11) and a PNP (Q12) that mimics the basic polarities of a PNP. At the same time, the overall effective is the product of NPN and PNP 's, allowing the combination to possess the good current drive characteristics of an NPN.

A welcome refinement is the ability to tolerate output short circuits to ground indefinitely, instead of the 709's "short period of time." This feat is accomplished by explicitly limiting the maximum output current to a sustainable value. Transistor Q15 is normally off, but if the op-amp attempts to source a current in excess of about 25mA, the drop across R11 causes Q15 to turn on. Doing so robs Q16 of base current, limiting further increases in output source current.

To protect the op-amp from *sinking* excessive current is a little more involved. If the voltage drop across R8 gets large enough, Q8 turns on and steals current from the base of Q9, thus ultimately limiting the output sinking current to a safe value. The LM101A tolerates short circuits to ground for *any* length of time.

Thanks to the current-source loads, two stages suffice to provide a nominal DC gain of over 500,000 (limited, in fact, by thermal feedback). The small number of stages (two) simplifies frequency compensation, which is provided by connecting a suitable network between the collector of Q4 and the pin labeled "comp." In many cases the network can be as simple as a single Miller compensation capacitor. Widlar's achievement is all the more remarkable for its having been accomplished with no computer simulation tools.

The chief difference between the 101A and the 101 is a modified input bias generator. The 101's

first-stage bias current is roughly constant, but the strong positive temperature coefficient of results in a base current with a strong negative temperature coefficient. To produce a more constant input current, Widlar biases the 101A's input stage with currents that are proportional to temperature. Transistors Q19-Q22 effectively form a thermometer to provide the desired behavior. As a bonus, the transconductance of the input stage, which is proportional to I_{bias}/V_T , also becomes much more temperature independent.

If R4 and R9 may be neglected for the moment, the voltage that appears across R1 is the difference between two pairs of diode voltages. Such a voltage is PTAT (proportional to absolute temperature), so the current through R1 would itself be PTAT if the resistance were stable over temperature.

Resistor R9 is added to reduce supply voltage sensitivity. As the supply voltage increases, the current through Q18 increases. The voltage at the base of Q22 would consequently increase, causing an undesired increase in amplifier bias currents. Inserting R9 provides an additional voltage drop that reduces the base voltage of Q22, thus offsetting the increase in Q18's current. Indeed, in the limit of very large current in Q18, the currents in Q21 and Q22 ultimately tend toward zero. A plot of output current vs. input current reveals a definite maximum (at an input current $V_T/R9$), so this type of current source is known as a "peaking current source." At the peak, the output current has a zero first-order sensitivity to input current. Centering the nominal input current about this peak provides supply-insensitive bias. Familiarity with the peaking current source is not nearly as widespread as it should be. The basic principle even works in CMOS technology.

"Easy to use" wins: The μ A741 (1968)

Back at Fairchild, Dave Fullagar had successfully debugged the 709's process problems. He learned of National's 101 and,

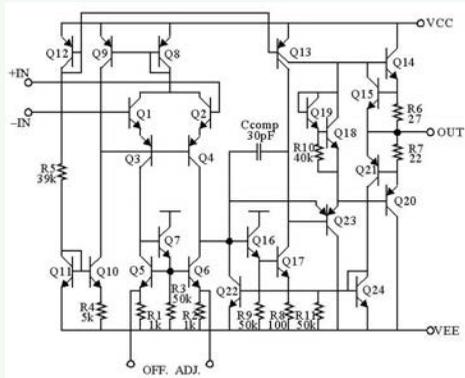


Figure 5: One variant of the 741

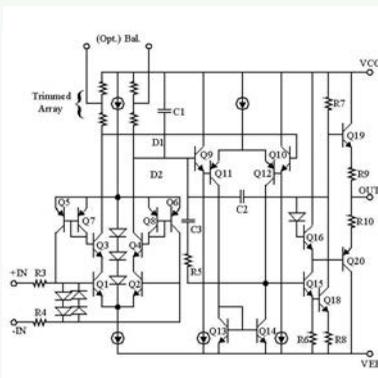


Figure 6: OP-07 (simplified)

according to his colleague George Erdi, wondered why it did not include an on-chip compensation capacitor. He felt that Fairchild's process technology could practically accommodate this goal, and speculated that National's could not yet do so [9]. His answer to the LM101 was the 741, the most popular op-amp of all time. Fullagar chose to retain the key architectural features of the LM101: the input stage is the same compound differential combination of an NPN emitter follower and PNP common-base amplifier, a mirror load provides high gain and single-ended conversion, and the second stage remains a current-source loaded common-emitter amplifier (Figure 5). A straightforward complementary emitter follower provides an output drive current that is limited to the same maximum value, and in the same asymmetrical way, as in the 101.

There are some differences, to be sure. Rather than using an elaborate replica bias circuit, the input stage employs simple feedback biasing to establish the base and collector currents. A Widlar mirror converts the $\sim 700\mu\text{A}$ master current in Q11, Q12 and R5 into a $20\mu\text{A}$ collector current in Q10. Transistors Q1 through Q4 act collectively as a single PNP transistor for common-mode inputs, and act together with Q8 and Q9 to form a Wilson mirror. The feedback connection of the Wilson mirror supplies the appropriate PNP base current automatically. George Wilson no doubt would have found the combination of the two mirrors amusing (if dissonant), as his own

invention (at Tektronix) was stimulated by the appearance of the Widlar mirror in the 709.

Those minor differences aside, the chief appeal of the 741 is its internal compensation capacitor. The popularity of the 741 validates Fullagar's implicit assumption that engineers are basically lazy (that is, very time-efficient). Engineers seem not to mind that a fixed capacitor degrades performance in most configurations. Ease of use, coupled with "good enough" performance, seems to be more important.

After a subsequent tenure at Intersil, Fullagar went on to co-found Maxim Integrated Products.

The Quest for Precision: The OP-07 (1975)

Although it is certainly true that Bob Widlar gets credit for a disproportionately large share of the analog IC innovations of the 1960s and 1970s, it would be terribly unfair to convey the impression that no one else was contributing to the development of the art. Fullagar's 741 and George Erdi's OP-07 from Precision Monolithics (now a part of Analog Devices) show that others were hard at work as well (Figure 6).

The three-stage OP-07 introduces two valuable techniques. One is the use of active base current cancellation. The other is trimming to reduce offsets by more than an order of magnitude over conventional approaches.

Current at the input terminals is reduced by well over an order of magnitude by measuring it, and

then supplying it internally. That is, the amplifier makes use once again of a special-purpose analog bias computer within the op-amp. Here, transistors Q3 and Q4 are not conventional cascodes at all. Rather, they are dummy devices whose sole purpose is to allow the measurement of base current. To the extent that the base currents of the cascoding transistors match those of the main input transistors (Q1/Q2), then the mirrors Q5/Q7 and Q8/Q6 will supply to the bases of Q1 and Q2 precisely the right amount of current. The external sources driving the op-amp input terminals only have to supply (or sink) the current resulting from incomplete cancellation.

The resistively loaded first stage contains numerous series-connected resistive segments, each having a reverse-biased junction in parallel with it. At wafer test time, the offset of the amplifier is measured, and an algorithm computes which resistive segment(s) should be shorted out to minimize the offset. Thanks to the magic of bipolar device physics, nulling out an offset this way tends also to minimize offset drift (if only it were so with MOS transistors). Then, a large current is passed through the corresponding reverse-biased junction, causing the aluminum metalization to spike through the junction and short out the resistor in question. Although it may seem that this brutal "zener-zapping" couldn't possibly be reliable, it allows the routine and robust attainment of sub- $100\mu\text{V}$ offsets, even if it's a bit rough on probe tips.

The second gain stage is a follower-driven PNP differential amplifier. Conversion to a single-ended output is performed the usual way, with an NPN mirror. Erdi bypasses around the slow PNP stage at high frequencies with R5 and C3, effectively turning the OP-07 into a two-stage op-amp where it matters. Miller compensation is provided around the remaining two stages with capacitor C2, and the dynamics of the overall amplifier are much like those of a 741 when all is said and done. The resistively-loaded common emitter third stage (Q18/R7)

drives a standard complementary emitter follower Q19/Q20 to complete the op-amp.

The combination of high gain, very low offset and low drift, coupled with 741-like dynamic behavior, assured the enduring popularity of this op-amp.

George Erdi left PMI in 1981 to co-found Linear Technology.

Voltage References

The op-amp may be the archetypal analog circuit, but it is certainly not the only important one. Voltage references are needed just about everywhere, if for no other purpose than to set supply and bias voltages to desired values. Data converters fundamentally require them also, if the mapping between bits and volts is to have any absolute quantitative meaning.

Somewhat ironically, the rising prominence of digital logic stimulated important advances in analog integrated circuits. As the digital revolution started to gain a head of steam in the late 1960s the need for regulated +5V supplies to power up the growing “gate farms” of TTL ICs became increasingly acute. Widlar foresaw a need for a simple, adjustment-free regulator chip, and set about to design it.

The problem, of course, is how to implement the fundamental voltage reference. Conventional alternatives, such as the zener diode, provide “reference” voltages after a fashion, but the actual voltages are not traceable to any reliable physics. The loose tolerances preclude realization of trim-free circuits without expensive component selection. A diode’s forward drop of “about 0.6V” might be somewhat more reliable (then again, maybe not), but the large negative temperature coefficient (of about 0.3%/°C) limits the useful temperature range.

As were most analog engineers, Widlar was well acquainted with a diode voltage’s large negative temperature coefficient. Rather than being stymied by it, however, he used this behavior as a starting point. A common rule of thumb is to expect about a 2mV drop in forward voltage for every degree Cel-

sus increase in temperature. The temperature coefficient is current-dependent (higher currents decrease its magnitude), but for a given fixed value of current, the temperature coefficient is nearly constant over an extremely wide temperature range. This type of behavior has been dubbed CTAT, for *complementary to absolute temperature*. More remarkable than this near-linear behavior is that the value of forward voltage extrapolated to absolute zero is the same for all diodes, and equal to the bandgap. The appearance of a voltage that is traceable directly to reliable physical constants (the bandgap, in this instance) is what makes trim-free voltage references possible.

To exploit these observations to make a voltage reference one must add a voltage that is PTAT (proportional to absolute temperature) to one that is CTAT. We’ve already witnessed Widlar’s familiarity with PTAT current sources, for they are part of the bias circuitry in the LM101A. Without putting too fine a

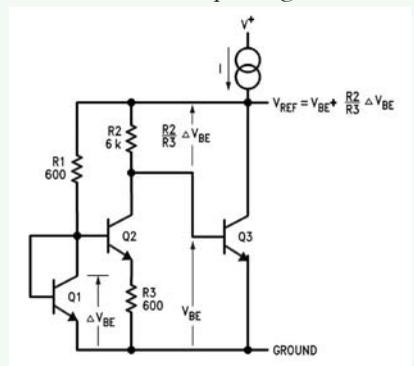


Figure 7: Simplified Widlar bandgap voltage reference

point on it, if you add a line that goes up, to a line that goes down, the sum will still be a line. And if the lines have equal and opposite slopes, the sum will be a constant. This flat condition occurs if the voltages sum to the bandgap voltage which, for silicon, is about 1.2V [we ignore in this discussion the second-order effects that one must consider when seeking to design a good bandgap reference].

Widlar’s translation of this recipe into circuit form appears in Figure 7 [11][12].

A Widlar current source (Q1 and Q2) establishes a voltage across R3

that is the difference between two junction voltages. The associated current is PTAT, as desired, but some scaling is needed to obtain the correct slope. To the extent that emitter and collector currents are substantially equal, the voltage across R2 is simply PTAT as well, but scaled by the ratio R2/R3. This voltage is added directly to the base-emitter voltage of Q3, so that the output reference voltage is the sum of a CTAT component (Q3’s VBE) and a PTAT one (a scaled ΔV_{BE}). When the ratio R2/R3 is chosen to produce an output voltage of about 1.2V at any one temperature, the output voltage remains very close to that value at all temperatures. This bandgap cell lies at the heart of Widlar’s LM109, the first three-terminal, trim-free voltage regulator IC. Variable volts may go in, but a constant 5V comes out. As a bonus, the LM109 offers both current limiting and thermal overload protection, making the part robust as well as easy to use.

The instant popularity of the LM109 speaks to the brilliance of Widlar’s particular implementation of the bandgap reference principle (and to his marketing insights). Nevertheless, Paul Brokaw of Analog Devices understood that the bandgap’s full potential remained to be realized. Brokaw set about systematically identifying effects that degrade performance. In bipolar transistors, collector current and base-emitter voltage are funda-

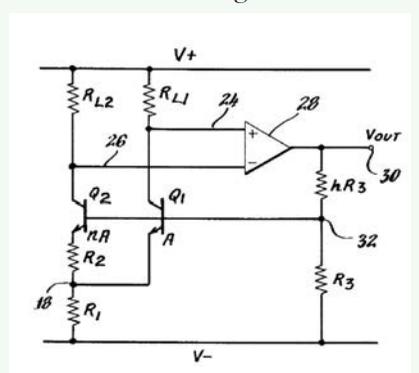


Figure 8: Brokaw bandgap cell [13]

mentally linked through dependable physics, but the Widlar cell depends on a secondary linkage between emitter and collector current, making it vulnerable to errors

of the second order. The simplified schematic of Figure 7 shows that nonzero base current is an important source of such errors

Within a short time, Brokaw devised an alternative implementation of the bandgap reference that does not suffer from these sensitivities (Figure 8):

This elegant circuit evades many of the second-order effects that degrade the Widlar cell's performance. Assume for simplicity that the collector load resistors $RL1$ and $RL2$ are equal. The negative-feedback loop involving the op-amp assures an equality of collector voltages, and thus, an equality of collector currents. The two transistors have unequal emitter areas, however, so the current *densities* are unequal. In turn, operation at different densities assures a calibrated difference in base-emitter voltages. This difference appears directly across $R2$, giving us a PTAT voltage, and an associated PTAT current through the resistor (and thus, through $Q2$).

The currents through $Q1$ and $Q2$ are equal and PTAT, and the current through $R1$ is therefore PTAT as well. From examination of the circuit, it should be clear that Brokaw has cleverly arranged for the common base connection to have a voltage expressible directly as the sum of a PTAT term (the voltage across $R1$), and a CTAT term (the base-emitter voltage of $Q1$). By ratioing $R2$ and $R1$ properly, the bandgap voltage appears at the base connection. If desired the feedback to the base from the op-amp can include the voltage divider shown, allowing the overall output voltage to be a multiple (here, $1+b$) of the bandgap voltage. The AD580 2.5V reference from Analog Devices has the distinction of being the first product to use the Brokaw bandgap cell, with the highest-accuracy versions offering total errors (including drift) of about 0.5% over the entire military temperature range.

The best-selling IC of all time

Almost every EE or hobbyist has encountered the 555 timer IC at some point, either in a lab class, or

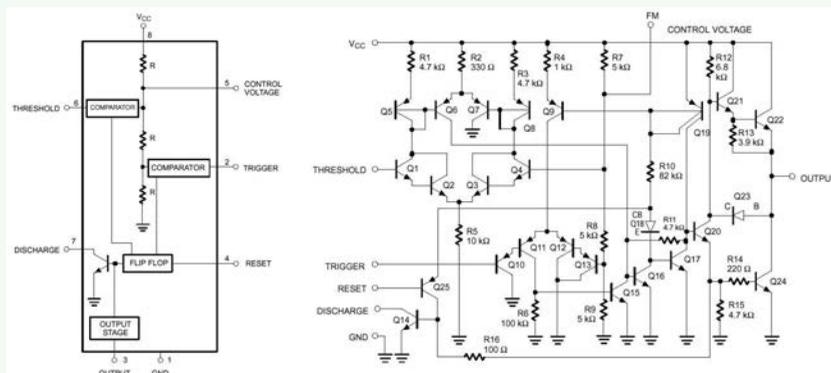


Figure 9: NE555 timer chip; block diagram and schematic (from the Philips Semiconductor datasheet)

“just hacking around.” Few of them, however, probably know much about who designed it, and how it came to be. And even fewer probably know that the 555 remains by far the best-selling integrated circuit in history, with about a billion units still sold each year, more than 35 years after its introduction.

The creator of the 555 is Hans Camenzind, who joined Signetics in 1968 with the intention of building the world's first integrated phase-locked loop (PLL). The NE565 PLL debuted in 1970, and quickly found widespread use in diverse applications. For example, the famous Altair 8800 computer that is often credited with kick-starting the PC revolution had an optional cassette tape interface for data storage. The two-tone frequency shift keyed (FSK) data was demodulated with a 565-based circuit. Today, PLLs are so widely used that it is hard to identify systems that don't have one or more of them. Camenzind's 565 went a long way toward converting PLLs from analog exotica into common functional blocks.

After putting the 565 into production, Camenzind took a leave of absence to write a book. He decided not to return to Signetics as a full-time employee but did agree to work for them on a contract basis. It was during this time that he designed the 555 timer chip (Figure 9). The proximate motivation was an outgrowth of his PLL work, with its recurrent need for stable, voltage-controlled oscillators. Camenzind expanded the scope of the project to make the chip a general-purpose timer IC.

No marketing study guided this decision. He needed such a component, and he simply assumed that others likely would, too [14].

Engineers quickly discovered that the particular complement of blocks chosen by Camenzind allows the 555 to perform a remarkably wide range of functions well beyond acting merely as a PLL adjunct. It is hard to imagine that any sort of marketing study would have resulted in its choice of two comparators, a flip-flop, a totem-pole output driver, and an open-collector transistor. And yet, somehow, this particular collection of analog atoms has enabled generations of engineers, hobbyists and tinkerers to create a rich variety of circuits and systems. Future archeologists, puzzled and intrigued by the seeming ubiquity of the 555, no doubt will conclude that it was the glue that held civilization together.

Bill and Dave, and the Wien bridge

The story of how Bill Hewlett and Dave Packard got their start is the stuff of legend. The two Stanford engineering students were encouraged by their advisor, Fred Terman (often known as “the father of Silicon Valley”), to found a company of their own. Hewlett had already designed an audio oscillator, and the pair chose that instrument as HP's first product. They dubbed it the model “200A” to mask the fact that it was the company's first product. A year later, in 1939, the sale of eight model 200B oscillators to Walt Disney Studios set the company on the path to history.

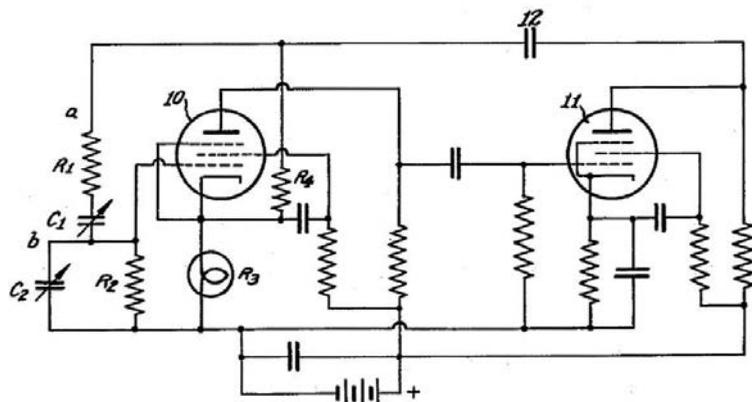


Figure 10: Hewlett's Wien-bridge oscillator [15]

Many engineers have made amplifiers oscillate by accident, so building an oscillator on purpose might seem easy. However Murphy guarantees that, much like washing your car in order to make it rain, things often don't work as desired. Hewlett's HP200 not only oscillates, but generates low-distortion sine waves over decades of frequency.

The core of the oscillator, not surprisingly, is a positive feedback loop (Figure 10); there is no net inversion in going from the control grid of vacuum tube 10 to the plate of vacuum tube 11.

A bandpass filter R1-C1-R2-C2 closes that feedback loop. Use of this type of RC network in a bridge configuration for measuring impedances was reported in 1891 by Max Wien (perhaps the most often misspelled name in the EE literature) [16]. The useful feature of this network here is the zero phase shift it exhibits at the bandpass filter's center frequency, allowing oscillation there. To produce a low-distortion sinusoidal output, the amplitude of this oscillation needs to be controlled by some mechanism. One could imagine an infinite variety of methods for doing so, but it's hard to imagine a more clever, elegant solution than Hewlett's: Monitor the amplitude with a *lightbulb*, and exploit the latter's thermal sensitivity to servo the amplitude to a controlled value. The oscillator thus has two feedback loops – a positive feedback loop to enable oscillation in the first place, and a negative feedback loop to stabilize the amplitude of that oscillation.

Resistors R4 and (lightbulb)

resistor R3 close the amplitude control loop. It is perhaps noteworthy that this loop uses current-mode feedback to the cathode of vacuum tube 10; the technique is therefore not nearly as modern as some seem to think. If the amplitude grows, the bulb's filament heats up, and the corresponding resistance increase causes the magnitude of the negative feedback to increase as well, opposing the amplitude increase. The nominal bulb current is so low that no visible glow is evident, and the bulb's lifetime typically well exceeds that of other components in the instrument.

The Gilbert and Jones Multipliers/Mixers

A circuit that resides at the intersection of analog computation and communication is the mixer. A multiplier is in fact a mixer; the choice of nomenclature is primarily a matter of context. When it comes to mixers (or multipliers), almost every communications engineer immediately thinks of the "Gilbert cell" or "Gilbert mixer." However most textbooks, and a great many journal and conference papers, actually describe an earlier invention by Howard E. Jones, instead of Barrie Gilbert's superficially similar multiplier (see Figures 11 and 12) [17][18].

The difference is seemingly trivial, but is in fact profound: Gilbert's brilliant insight is that representing variables entirely in the current domain can enable spectacular linearity, despite the famous exponential nonlinearity of bipolar transistors. The fundamental idea may be viewed as employ-

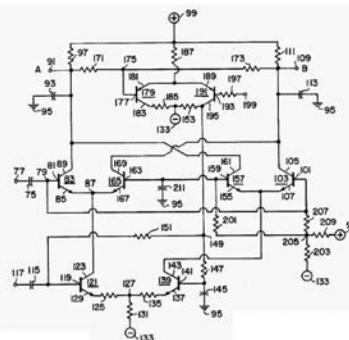


Figure 11: Circuit due to Jones [17]; input and output quantities are voltages

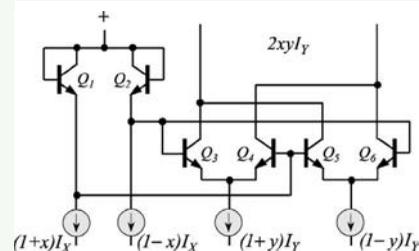


Figure 12: Gilbert cell example (with predistorting pair Q1/Q2); variables are currents [18]

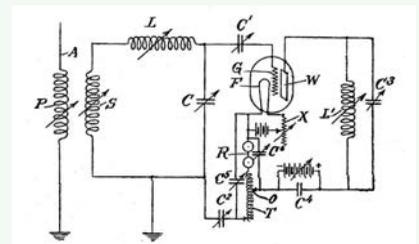


Figure 13: Armstrong's first great invention: Regeneration [19]

ing predistortion to undo precisely the inherent nonlinear transfer characteristics of the core circuitry. The reliable nature of a bipolar device's nonlinearity enables such cancellation to succeed.

Despite earnest attempts by Gilbert himself to correct this misapprehension, it is hard to undo decades of error overnight [18].

Analog Electronics in Communications

What we call electronics today was once mainly wireless. The Widlar of wireless was Edwin Howard Armstrong, who explained to Lee de Forest, inventor of the triode vacuum tube how it actually worked, and then exploited that understanding to invent circuits and systems that still dominate today. Those contributions include

the superheterodyne receiver (which popularized the mixer), and wideband FM. His first important discovery, however, was the boost in amplification provided by positive (“regenerative”) feedback, using circuits such as shown in Figure 13 [19].

This circuit uses a transformer T to couple signals from the plate (“wing,” W) back to the cathode (filament, F). As far as this feedback loop is concerned, the vacuum tube operates as a (non-inverting) common-grid amplifier, and so the connection constitutes a gain-boosting positive feedback amplifier.

Thanks to regeneration, the vacuum tube transformed from an expensive, erratic curiosity into the very basis for a new field – electronics. Early vacuum tubes struggled to evince voltage gains of five when used without feedback, but regeneration enabled arbitrary gains – even oscillation. For the first time, engineers had fully electronic high-gain amplifiers and compact oscillators at their disposal, allowing electrical engineering to move rapidly beyond its power-engineering origins. Positive feedback’s importance is underscored by the difficulty of engineers to appreciate the value of negative feedback. The idea of throwing away precious gain seemed absurd to a generation of engineers who had enjoyed high gain for the first time. Paradigm shifts of that magnitude take time.

Armstrong’s invention of the superheterodyne receiver in the closing days of the first World War is all the more remarkable for its overwhelming dominance even as it approaches its 90th year. Unprecedented ease of operation conferred by the single required tuning control, coupled with circuit improvements and cost reductions made possible by better vacuum tubes, made the superhet the dominant architecture by 1930. Generations of engineers have never known another.

Modesty wins again

Decades after Armstrong’s invention of the superhet, and decades

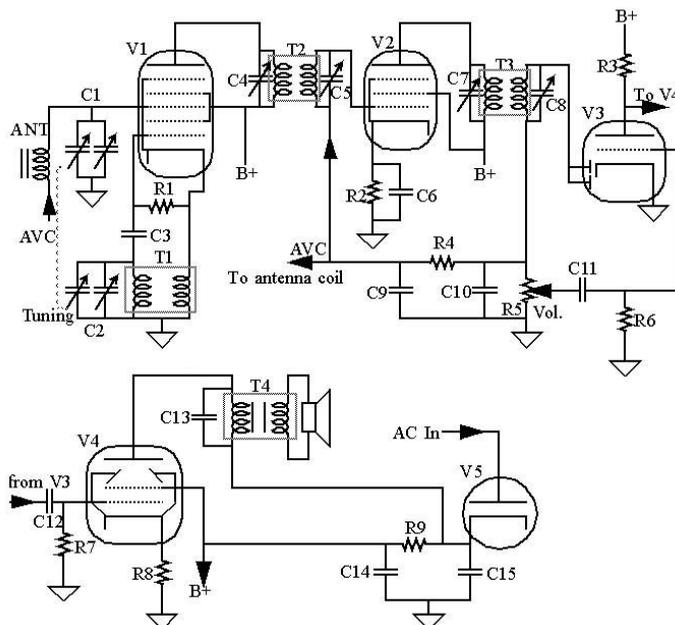


Figure 14: The “All-American Five”

before the iPod, there was the “All-American Five.” For about thirty years, this AM receiver was the most popular tabletop radio. The All-American’s complement of five vacuum tubes kept costs low, while delivering satisfactory performance (Figure 14).

The first tube is a pentagrid converter, which acts as both local oscillator and mixer. In some sense, it may be viewed as an early integrated circuit. The local oscillator part of this circuit is an echo of Armstrong’s original regenerative oscillator. The cathode current couples back to the first grid through transformer T_1 whose tuned secondary controls the oscillation frequency and, therefore, the channel selected, as in all superhets. Simultaneous tuning of a simple bandpass filter at the RF input port aids image rejection. The tuning capacitors for both circuits are mechanically linked (“ganged”) so that the consumer only has to turn one knob to change frequency.

Grid 2 is incrementally grounded, and acts as a Faraday shield to isolate the oscillator and RF circuits. The RF signal feeds grid 3, and nonlinear interaction within the tube performs the mixing action. Grids 4 and 5 are incrementally grounded, and remove the Miller effect and suppress secondary electron emission, respectively.

The output of the first stage is coupled through a doubly-resonant IF bandpass filter to a single IF amplifier, a 12BA6 (V2), operating at 455kHz. The 12BA6 is a pentode, and thus behaves much like a cascode, allowing one to use filters on both the input and output ports without worrying about detuning or instability from feedback.

Demodulation and audio amplification take place in V3, a 12AV6, which contains two diodes and a triode within one glass envelope. The diodes perform envelope detection, and the triode amplifies the demodulated audio. The demodulated output in turn feeds two destinations. One is the output power amplifier, V4. The other is an additional low-pass filter, the output of which is the average of the demodulated output. This signal is used to control automatically the gains of the front-end and IF amplifier as a function of received signal strength. The greater the demodulated output, the more negative the bias fed back to those stages, reducing their gain. This automatic gain control (AGC) or automatic volume control (AVC) thus reduces potentially jarring variations in output amplitude as one tunes across the dial.

V4 is a 50C5 beam-power tube used in a Class A audio power amplifier configuration. Trans-

former coupling provides the necessary impedance transformation to deliver roughly a watt of audio into the speaker.

A 35W4 (V5) power rectifier generates the B+ plate supply for the other tubes.

With minor variations, the All-American was widely copied, and clones could be found all over the world. Once it caught on, high manufacturing volumes drove down the cost of these five particular tube types, so anyone designing a new radio intended for a cost-sensitive application tended to use the same tubes, and thus used similar circuits. Variations among different versions are really quite slight (e.g., small resistor or capacitor value differences, absence or presence of cathode resistor bypass capacitors, etc.), and the basic schematic of Figure 14 suffices for most troubleshooting purposes.

Early Personal Audio: The Regency TR-1 Transistor Radio

The first portable transistor radio became available in time for Christmas in 1954 and was the result of a conscious effort by a young Texas Instruments to create a mass market for transistors. Up to this time, the only commercial use for transistors had been in hearing aids. As the father of the project, Patrick Haggerty, later noted, the thinking was that "...a dramatic accomplishment by [us would] awaken potential users to the fact that...we were ready, willing, and able to supply [transistors]" [20]. TI arranged a deal with a small company called IDEA (Industrial Development Engineering Associates), whose Dick Koch modified TI's first-pass circuit (principally designed by Paul D. Davis and Roger Webster) to reduce cost and improve manufacturability. The task was challenging as no one had much expertise with transistors yet. To make a tough job even more difficult, the germanium transistors then available were quite poor by today's standards (f_T 's of only a few MHz at best, and β 's of 10-20), while their cost was high. Compounding those difficulties was the lack of off-the-shelf minia-

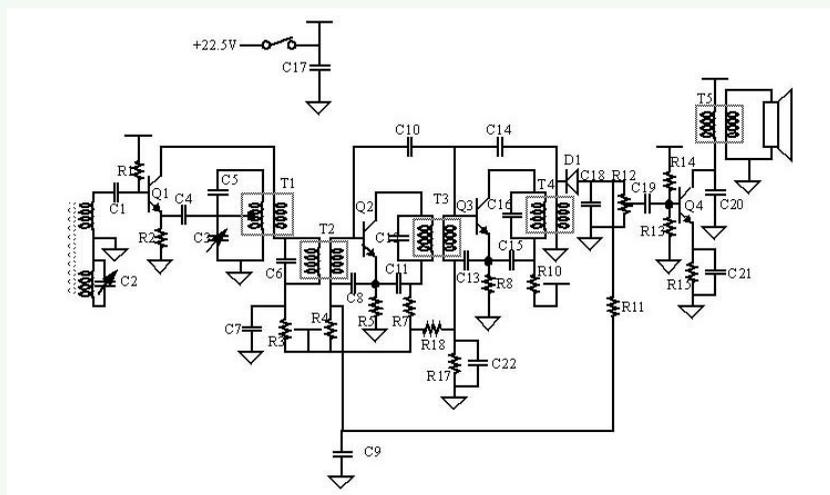


Figure 15: Schematic of the Regency TR1 [20]

ture components to complement the small transistors. It was quite a struggle to cram all of the circuitry into a case small enough to fit in a shirt pocket (indeed, early advertisements used a custom-made shirt with oversized pockets). In a first for consumer electronics, printed-circuit technology was chosen for the TR1 in order to facilitate interconnecting such densely packed circuit elements. The newness of the technology presented many daunting manufacturing challenges.

Calculations showed early on that no more than four transistors could be used or IDEA and its Regency division would not be able to make a profit at the targeted sale price of \$49.95. The four transistors accounted for about half of the cost of the materials. At a time when an All-American Five could be purchased for about \$15, it was difficult to imagine that there would be a significant market for such an expensive device. As it happened, demand outstripped production capacity for quite some time.

As seen in Figure 15, four transistors were enough. In this circuit, the first transistor, Q1, functions as an oscillator-mixer, just as the first tube does in an All-American Five. Transformer coupling between collector and emitter circuits provides the positive feedback necessary for oscillation.

The incoming RF signal is tuned using a mechanism called "absorption," developed by the German

company Telefunken around World War I. In this technique, an LC tank coupled to the input circuit shorts out (absorbs) signals at all frequencies other than the resonant frequency of the tank. The RF signal can pass to the base of Q1 only when this shorting disappears, at the absorbing tank's resonant frequency (determined by C2). The inherent nonlinearity of the base-emitter diode provides the mixing action. Hence, in addition to the local oscillator signal, the collector current also has a component at the sum and difference heterodyne terms. The difference signal is then fed to the first IF amplifier, Q2, through an LC bandpass filter tuned to the IF of 262kHz. The unusually low IF allows the low- f_T transistors to provide useful amounts of gain, but exacerbates an already bad image rejection problem. The variable capacitor in the absorptive LC front-end tank is ganged with the LO variable capacitor. The degree of image rejection achieved here is best described as adequate.

The second IF amplifier, Q3, is connected in a manner essentially identical to Q2. The large C_{μ} values (probably about 30-50 pF) are partially cancelled by positive feedback through C10 and C14 (a technique introduced in the 1920's as the Neutrodyne circuit).

A standard envelope detector performs demodulation, and then feeds a single stage of audio amplification. Transformers couple signals into the detector and out of

the audio amplifier.

AGC action is provided in a familiar manner: the demodulated audio is further RC filtered (here by R_{11} and C_9), and the resulting negative-polarity feedback signal controls the gain of the first IF stage by varying its bias.

The success of the TR-1 had important consequences beyond establishing TI as a leader in the semiconductor business. Of particular significance is that IBM quickly abandoned development of new vacuum tube computers, with Thomas Watson, Jr. reasoning that if transistors were mature enough for high-volume consumer gear, it was time to consider them for computers. As he later told the story, every time one of his subordinates expressed doubt about transistors, he'd give him a TR-1, and that usually settled the argument [21].

A young company called Sony introduced their own transistor radio, the TR55, soon after Regency's TR1 debuted. The company would soon dominate the consumer market for portable electronics.

Sophisticated Low Tech: Three-Transistor Toy Walkie-Talkie

Although vacuum-tube "toy" walkie-talkies had appeared as hobby projects in the years following the second World War, they were too expensive for anyone to consider manufacturing them in volume as actual toys for children. The development of the transistor made such a toy a practical possibility. Jerry Norris, an engineer at Texas Instruments, was the first to act on this insight, and in so doing developed in 1962 the ancestor of all toy walkie-talkies [22]. This widely copied and ingenious circuit uses a single-transistor *superregenerative* amplifier/detector (yet another Armstrong invention), followed by two stages of audio amplification in receive mode (see Figure 16). When transmitting, the superregenerative stage becomes a stable crystal-controlled 27MHz oscillator, amplitude-modulated by an audio amplifier built out of the other two transistors. The speaker doubles as a micro-

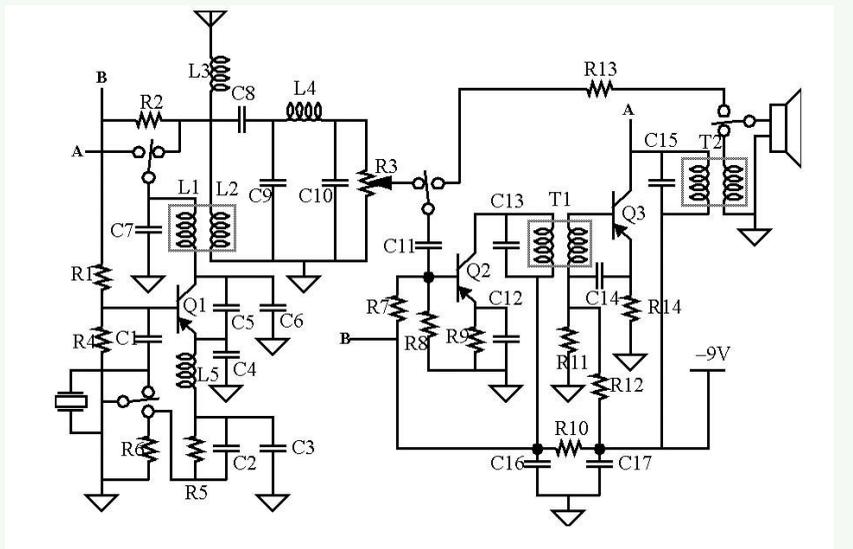


Figure 16: Jerry Norris' superregenerative CB walkie-talkie [22]

phone in this mode.

Transistor Q1 does all the RF work in this circuit. In receive mode, Q1 is configured as a Colpitts oscillator with *unstable* bias. An incoming RF signal establishes an initial condition from which oscillations build up exponentially, providing remarkable sensitivity. The bias is arranged to cut off ("quench") the oscillations periodically at a rate high enough to sample the modulation at a super-Nyquist rate (this periodically quenched oscillation distinguishes superregeneration from regeneration). Thanks to ever-present nonlinearities, the transistor also amplifies the modulated RF signal asymmetrically. Hence, the collector current contains a component roughly proportional to the modulation itself. A low-pass filter consisting of C_9 , L_4 and C_{10} removes the RF component, passing only the modulation to the two-transistor audio amplifier made of Q2 and Q3.

While Q1 acts as a self-quenched LC oscillator in the receive mode, a quartz crystal is used to control the frequency of oscillation during transmit. Resistor R_6 is shorted out during transmit to prevent quenching.

The oscillator amplitude is roughly proportional to the collector supply voltage, so varying the supply voltage with an audio signal from Q2/Q3 amplitude-modulates the carrier. Although the dis-

tortion from this process hardly meets the standards of high fidelity audio, it is certainly adequate for voice communications, and most definitely adequate for a toy.

Because this simple circuit provides such large gain with so few transistors, it continues to dominate the toy walkie-talkie market, having been copied and modified countless times by manufacturers. The influence of Norris' circuit is evident from having traced over twenty superregenerative walkie-talkie circuits over the years. In all of them, just one transistor does all of the RF work, with the remaining two (sometimes three) transistors serving as audio amplifiers. As with the All-American Five, variations among different manufacturers are relatively minor.

The world is analog

Although it is perhaps a little ironic that the story of linear circuits itself seems to have been so nonlinear, a linear narrative would have been a distortion of history (beyond those already committed). As the world allegedly "goes digital," these histories help remind us that we live in an analog world, after all.

Analog epilogue: A bit more about Widlar

Widlar essentially created the analog IC business, and so perhaps it

is appropriate to say a little more about him in this sidebar.

The individualism evident in his circuit designs reflects his independent, idiosyncratic personality. While still at Fairchild, he acquired a reputation as a hard-working, hard-drinking prankster. By the time he'd joined National Semiconductor, his antics were well on their way to becoming legendary, as is evident from a sidebar accompanying an August 1968 article by him in *EEE* ("Bob Widlar of National Semiconductor speaks out on what makes a good IC").

He was famous for total immersion when working on a design. He could work nonstop into a state of such exhaustion that he found relief by driving his beloved '66 Mercedes 280SL convertible to the airport and purchasing a ticket for "the next flight out."

The mere fact of his having a gun collection might have made some of his colleagues a bit nervous; knowing that he used, for target practice, beer cans with the names of those not in his esteem probably unnerved the rest.

The reporter who interviewed him noted that Widlar's apartment was stocked only with scotch, beer and glasses. "His refrigerator is bare if you don't count the ice cubes." This comment only hinted at the magnitude of Widlar's ability to imbibe.

When National, along with the rest of the electronics industry, suffered during a recession a couple of years later, the groundskeeping staff was eliminated as part of a corporate cost-cutting plan. Widlar didn't like the unkempt look of the facilities as the weeds grew. His response was to drive with Bob Dobkin to someplace south of San Jose and purchase a sheep (some say it was a goat, but look at the photo in Figure 17 and decide for yourself). Upon returning to National, "someone" called a reporter at the San Jose Mercury News, and a photographer appeared soon after to document National's new lawn-mowing technology in action.

The groundskeeping staff was rehired soon afterwards.

Later that day, Widlar took the



Figure 17: Bob Widlar (standing over a chip plot of the LM10); the infamous groundskeeping sheep (with a bemused National employee, Vickie Darst, looking on). Both photos courtesy of Bob Pease and the National Semiconductor Archives.

sheep with him to Marchetti's, a popular National watering hole in those days. He left it with the bartender. History does not record what the bartender did with the sheep.

Only a few years after joining National, Widlar's stock options had appreciated sufficiently (thanks in large part to his designs) that he "retired" from National Semiconductor at about 10:30 PST, 21 December 1970. Not long after, he drove his Mercedes down to Puerto Vallarta, Mexico, where he lived the rest of his life. He had celebrated his 33rd birthday just the month before.

After a brief period of time in which he worked with a fledgling Linear Technology (co-founded by Bob Dobkin of the sheep adventure), he returned to designing for National Semiconductor on a contract basis. During this time he designed an op-amp (the LM10) that delivered 741-like specifications while operating off of a single 1.2V supply. If that weren't impressive enough, he included a bandgap voltage reference (the reader will note that the nominal supply voltage does not exceed a bandgap voltage). He followed that achievement with the LM11, a bipolar op-amp with 25pA input bias current. His next design represented a leap from one power extreme to the other: The LM12 is an operational amplifier capable of

10-ampere output currents and 80W continuous dissipation (800W peak). Its integral protection is so comprehensive that considerable effort is required to destroy it.

After a life of extreme habits, he eventually adopted a healthier lifestyle, and began jogging regularly. On one of these jogs in early 1991, he suffered a fatal heart attack near his home in Puerto Vallarta. He was only 53 years old.

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